A vector from the origin to \((a, b, c)\) is parallel to the line in question, and by hypothesis, \(a^2 + b^2 + c^2 = 1\).

[How would you work this if I don't choose this point — i.e. \(a^2 + b^2 + c^2 \neq 1\)]

From \(\vec{r} \cdot \hat{u} = \|\vec{r}\| \|\hat{u}\| \cos \theta_x = \cos \theta_x\),

\(\vec{r} \cdot \hat{u} = a\),

we get \(a = \cos \theta_x\).

Show the work to get

\(b = \cos \theta_y, \; c = \cos \theta_z\).

So \((\cos \theta_x)^2 + (\cos \theta_y)^2 + (\cos \theta_z)^2\)

\(= a^2 + b^2 + c^2 = 1\).

Again, what if \(a^2 + b^2 + c^2 \neq 1\)?
(3) First, look at row 1:

\[ \cos \Theta_{xx} \cos \Theta_{yy} \cos \Theta_{zz} \]

or \[ a_{11} \quad a_{12} \quad a_{13} \]

or \[ l, \quad m, \quad n \]

These are the direction cosines of the line \( f(x, y, z) = x' \) or the vector \( \hat{v} = x' \).

So, from problem 2, we know that a vector made from these:

\[ \overrightarrow{v}' = (\cos \Theta_{xx}', \cos \Theta_{yy}', \cos \Theta_{zz}') \]

is normalized.

(4) A similar argument can be made for the other two rows, and all 3 columns. So do that — if it's easy, it shouldn't be hard.
Now, make vectors of row 1 and row 2

\[ \hat{x}' = (\cos \Theta_{xx}', \cos \Theta_{yx}', \cos \Theta_{zx}') \]

\[ \hat{y}' = (\cos \Theta_{xy}', \cos \Theta_{yy}', \cos \Theta_{zy}') \]

or

\[ \hat{x}' = (a_{11}, a_{12}, a_{13}) \]

\[ \hat{y}' = (a_{21}, a_{22}, a_{23}) \]

The dot product of these two is:

\[ \cos \Theta_{xx}' \cos \Theta_{xy}' + \cos \Theta_{yx}' \cos \Theta_{yy}' + \cos \Theta_{zx}' \cos \Theta_{zy}' \]

or

\[ a_{11} a_{21} + a_{12} a_{22} + a_{13} a_{23} \]

Now construct the 1,2 (row 1, column 2) element of the product $AA^T$
3. and you should get:

\[ \cos \theta_{xx} \cos \theta_{yy} + \cos \theta_{yx} \cos \theta_{xy} + \cos \theta_{xx} \cos \theta_{yy} \]

\[ = a_{11} a_{21} + a_{12} a_{22} + a_{13} a_{23} \]

Note that this is the same as the dot product we calculated.

AND since the matrix we started with is orthogonal, the \((1,2)\) element of \(AA^T\) is zero. \[\text{Q.E.D.}\]

It should be equally straightforward to show for the other pairs of rows and columns ...
(4) OK - I'm supposed to use prob 3 to work prob 4, but I (effectively) used prob 4 to work 3. So I'm in a pickle. I'll need another way to show that:

$$AA^T = A^TA = 1.$$ 

So, consider the vector

$$\vec{x} = (x, y, z) = (0, 1, 3).$$

From eqn. 2.12

$$\vec{x}' = (x', y', z') = a_{ij} x_j = \begin{pmatrix} a_{11} x_1 \\ a_{21} x_2 \\ a_{31} x_3 \end{pmatrix} = (a_{11}, a_{21}, a_{31}).$$
From 2 we know this is normalized. And if I construct the \((1,1)\) element of \(AA^T = IA\)

I get

\[
\begin{bmatrix}
q_{11} & q_{21} & q_{31} \\
q_{12} & q_{22} & q_{32} \\
q_{13} & q_{23} & q_{33}
\end{bmatrix}
\begin{bmatrix}
q_{11} \\
q_{21} \\
q_{31}
\end{bmatrix}
\]

\[
= q_{11}q_{11} + q_{21}q_{21} + q_{31}q_{31}
\]

\[
= q_{i1}q_{i1} = 1
\]

You can do the same starting with \(\vec{v} = (\vec{y}, \vec{z}, \vec{b})\), etc. and show that the diagonal elements of \(A^TA\) are all 1.
so I've shown that the columns of $A$ are normalized.

Starting with $\bar{x}_y = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Eqn 2.12 gives:

$$\bar{x}_y' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = a_{ij} x_j = \begin{pmatrix} a_{11} x_1 \\ a_{21} x_1 \\ a_{31} x_1 \end{pmatrix} = \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}$$

If I now construct $\bar{x}_x', \bar{y}_y'$, I get

$$\begin{pmatrix} a_{11} a_{21} a_{31} \\ a_{11} a_{22} \\ a_{11} a_{32} \end{pmatrix} = a_{11} a_{12} + a_{21} a_{22} + a_{31} a_{32}$$

This is close to the $(1,2) = (\text{row 1, column 2})$ element of $A^T A$.
It is also the dot product of the vectors made from the first and second columns of $A$.

Finally, I note that since

$$\vec{r}_x \cdot \vec{r}_y = 1 \cdot 1 = 0$$

then $a_{11}a_{12} + a_{21}a_{22} + a_{31}a_{32} = 0$

so the 1st and 2nd columns of $A$ are orthonormal vectors.

Consider $\vec{z} = (\vec{0})$ to finish up the proof.

Using vectors like $\vec{z}' = (x'), (y'), (z')$, you can show that the rows of $A$ are an orthonormal basis.
AND at long last

sin the diagonal elements

of $A^T A$ are 1 and the

offdiagonal elements are 0,

we have $A^T A = 1$, so

$$A^T = A^\dagger$$

$A$ is orthogonal.

Aside: $A^T A$ involves multiplying columns of $A$

by each other.

Since $A A^T$ involves multiplying

rows of $A$ by each other, if you do as

the previous page says, you'll be able to

argue that $A A^T = 1$
8) Since $u'_k = a_{ki} u_i$ and $v'_e = a_{je} v_j$,

in class we showed that if

$$u'_k = a_{ki} u_i,$$

then $u_k = a_{ik} u'_i$

so $u_k v'_e = a_{ik} u'_i a_{je} v'_j$

$$= a_{ik} a_{je} u'_i v'_j$$

or

$$T_{ke} = a_{ik} a_{je} T_{ij}$$

= Note where the indices are!

Maybe this looks less like mindless index shifting if we do a specific examples.

(So this is all log manip - we just did prob)
Example - let \( U \) and \( V \) be two orthonormal vectors in the \( xy \) plane:

\[
U = \begin{pmatrix} \cos x \\ \sin x \\ 0 \end{pmatrix}, \quad V = \begin{pmatrix} \sin x \\ -\cos x \\ 0 \end{pmatrix}
\]

[Can you prove they're orthonormal?]

Now, rotate the system an angle \( \theta \) around the \( z \) axis:

\[
A = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Construct the outer product:

\[
UV = \begin{pmatrix} \cos x \\ \sin x \\ 0 \end{pmatrix} \begin{pmatrix} \sin x & -\cos x & 0 \end{pmatrix}
\]

\[
T_{ij} = \begin{bmatrix} \sin x \cos x & -\cos^2 x \\ \sin^2 x & -\sin x \cos x \\ 0 & 0 & 0 \end{bmatrix}
\]
Now, X-form $u$ and $v$ to

$$u' = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha \\ \sin \alpha \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \alpha + \sin \theta \sin \alpha \\ -\sin \theta \cos \alpha + \cos \theta \sin \alpha \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha - \theta) \\ \sin(\alpha - \theta) \\ 0 \end{bmatrix}$$

- does this make sense?

Note also that $|u'|^2 = |u|^2$

$$v' = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin \alpha \\ -\cos \alpha \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \theta \sin \alpha - \cos \alpha \sin \theta \\ -\sin \theta \sin \alpha + \cos \alpha \cos \theta \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sin(\alpha - \theta) \\ -\cos(\alpha - \theta) \\ 0 \end{bmatrix}$$
Now construct the outer product of \( \mathbf{u}' \) and \( \mathbf{v}' \):

\[
\mathbf{u}' = \begin{bmatrix}
\cos(\alpha - \Theta) & \sin(\alpha - \Theta) & 0 \\
\sin(\alpha - \Theta) & -\cos(\alpha - \Theta) & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\mathbf{T}'_{ij} = \begin{bmatrix}
\cos(\alpha - \Theta)\sin(\alpha - \Theta) & -\cos^2(\alpha - \Theta) & 0 \\
\sin^2(\alpha - \Theta) & -\cos(\alpha - \Theta)\sin(\alpha - \Theta) & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

So the last step is to confirm that \( \mathbf{uv} \) transforms to \( \mathbf{u'}v' \) properly:

\[
\mathbf{u'}\mathbf{v'} = q_i q_j \mathbf{u_i} \mathbf{v_j}
\]
So, for example:

\[ u_i' = a_{ij} v_j \]

\[ = a_{i1} a_{11} u_{11} v_{11} + a_{i1} a_{12} u_{12} v_{12} + a_{11} a_{13} u_{13} v_{13} \quad i = 1 \]
\[ + a_{i2} a_{21} u_{21} v_{11} + a_{i2} a_{22} u_{22} v_{22} + a_{21} a_{23} u_{23} v_{23} \quad i = 2 \]
\[ + a_{i3} a_{31} u_{31} v_{11} + a_{i3} a_{32} u_{32} v_{22} + a_{31} a_{33} u_{33} v_{33} \quad i = 3 \]

\[ j = 1 \quad j = 2 \quad j = 3 \]

\[ = \cos^2 \theta \cos \phi \sin \alpha + \cos \theta \sin \theta (- \cos^2 \alpha) + \cos \theta (0) u_{13} \]
\[ + \sin \theta \cos \theta \sin^2 \alpha + \sin^2 \theta (- \cos \phi \sin \alpha) + - 0 - \]
\[ + \text{all zeros} \]

\[ = \cos^2 \theta \sin \alpha \cos \alpha - \sin \theta \cos \theta \cos^2 \alpha + \sin \theta \cos \theta \sin^2 \alpha - \sin^2 \theta \sin \alpha \cos \alpha \]
\[ = \cos \theta \cos \alpha \left[ \cos \theta \sin \alpha - \sin \theta \cos \alpha \right] \]
\[ + \sin \theta \sin \alpha \left[ \cos \theta \sin \alpha - \sin \theta \cos \alpha \right] \]
\[ = \left[ \cos \theta \cos \alpha + \sin \theta \sin \alpha \right] \left[ \cos \theta \sin \alpha - \sin \theta \cos \alpha \right] = \cos (\theta - \alpha) \sin (\alpha - \theta) \]
Let's see if anything interesting happens for $T'_{13}$:

$$u_1' v_3' = a_{1i} a_{3j} u_i v_j$$

For $j = 1, 2, 3$:

$$j = 1$$

$$= a_{11} a_{31} u_1 v_1 + a_{11} a_{32} u_1 v_2 + a_{11} a_{33} u_1 v_3 - i = 1$$

$$+ a_{12} a_{31} u_2 v_1 + a_{12} a_{32} u_2 v_2 + a_{12} a_{33} u_2 v_3 - i = 2$$

$$+ a_{13} a_{31} u_3 v_1 + a_{13} a_{32} u_3 v_2 + a_{13} a_{33} u_3 v_3 - i = 3$$

$$= 0$$ since every term a term $a_{3i} a_{1i}$, which are always zero (so no, nothing interesting happened)

**You do $T'_{21}$ and $T'_{31}$ (at least you should do them all!**
2. The 4th eqn is \[ \frac{\partial u}{\partial x_i} \frac{\partial x_i}{\partial x_j} \]

So I sum on \( j \) and \( i = 1, 2, 3 \) for each.

So we really have 3 expressions, each of which has 3 terms:

\[ \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial x_1} + \frac{\partial u}{\partial x_2} \frac{\partial x_2}{\partial x_1} + \frac{\partial u}{\partial x_3} \frac{\partial x_3}{\partial x_1} \]

\[ \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial u}{\partial x_2} \frac{\partial x_2}{\partial x_2} + \frac{\partial u}{\partial x_3} \frac{\partial x_3}{\partial x_2} \]

\[ \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial x_3} + \frac{\partial u}{\partial x_2} \frac{\partial x_2}{\partial x_3} + \frac{\partial u}{\partial x_3} \frac{\partial x_3}{\partial x_3} \]

A mathematician would recognize this as the chain rule where I'm considering that \( x'_i = f(x_1, x_2, x_3) \).

So I get the variation of \( u \).
2cont'd with respect to $x_1$, by considering how much $u$ varies as I vary $x_1$ 
\[
\left( \frac{\partial u}{\partial x_1} \right)
\] times the rate at which $x_1$ varies as I vary $x_1'$: 
\[
\left( \frac{\partial x_1}{\partial x_1'} \right)
\]
But I can also vary $x_1'$ by varying $x_2$. So there will be a term like:
\[
\left( \frac{\partial u}{\partial x_2} \right) \left( \frac{\partial x_2}{\partial x_1'} \right)
\]
And the total change in $u$ because of a change in $x_1'$ will be:
\[
\frac{\partial u}{\partial x_1'} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial x_1'} + \frac{\partial u}{\partial x_2} \frac{\partial x_2}{\partial x_1'} + \frac{\partial u}{\partial x_3} \frac{\partial x_3}{\partial x_1'}
\]
Consider an analogy of a hill with an $xy$ coordinate system. A contour plot might look like:
If I start at some point \((x_0, y_0)\) and move to a nearby point \((x_0 + \Delta x, y_0 + \Delta y)\) the altitude is likely to change. By how much does my altitude \(Z(x, y)\) change? If I imagine that I do this in two steps, I calculate:

\[
\Delta Z(x, y) = Z(x + \Delta x, y) - Z(x, y) = \frac{Z(x + \Delta x, y) - Z(x, y)}{\Delta x} \cdot \Delta x
\]

\[
= \frac{dZ(x, y)}{dx} \Delta x
\]

for the first step along \(x\), plus...
\[ \Delta z(x, y) = z(x + \Delta x, y + \Delta y) - z(x + \Delta x, y) \]
\[ = \frac{z(x + \Delta x, y + \Delta y) - z(x + \Delta x, y)}{\Delta y} \cdot \Delta y \]
\[ = \frac{\partial z(x, y)}{\partial y} \Delta y \]

But suppose I transform my axes somehow \([x' = f(x, y)]\) so that I can do it in one step along \(x'\) (of size \(\Delta x'\)). Then the slope of the hill along \(x'\) will be

\[ \frac{\Delta z}{\Delta x'} = \frac{\partial z}{\partial x} \frac{\Delta x}{\Delta x'} + \frac{\partial z}{\partial y} \frac{\Delta y}{\Delta x'}, \quad \text{so} \]

\[ \frac{\partial z}{\partial x'} = \frac{\partial z}{\partial x} \frac{\Delta x}{\Delta x'} + \frac{\partial z}{\partial y} \frac{\Delta y}{\Delta x'} \]

= returning to the problem =
we have \[ x_1 = a_{11} x'_1 + a_{21} x'_2 + a_{31} x'_3 \]
\[ = a_{i1} x'_i \]
\[ x_2 = a_{i2} x'_i \]
\[ x_3 = a_{i3} x'_i \]
and so \[ x'_j = a_{ij} x'_i \]

\[ \frac{\partial x_1}{\partial x'_1} = a_{11} \quad \frac{\partial x_1}{\partial x'_2} = a_{21} \]
\[ \frac{\partial x_2}{\partial x'_1} = a_{12} \quad \frac{\partial x_2}{\partial x'_3} = a_{13} \]

so \[ \frac{\partial x'_i}{\partial x'_j} = a_{ij} \]

Putting this into \[ \frac{\partial u}{\partial x'_1} = \]
\[ \frac{\partial u}{\partial x'_1} = \frac{\partial u}{\partial x'_1} a_{11} + \frac{\partial u}{\partial x'_2} a_{12} + \frac{\partial u}{\partial x'_3} a_{13} = a_{ij} \frac{\partial u}{\partial x'_j} \]
So the gradient in $x''$ is:

$$
\nabla' u = \frac{\partial u}{\partial x'_1} + \frac{\partial u}{\partial x'_2} + \frac{\partial u}{\partial x'_3}
$$

$$
= a_{1j} \frac{\partial u}{\partial x_j} + a_{2j} \frac{\partial u}{\partial x_j} + a_{3j} \frac{\partial u}{\partial x_j}
$$

$$
= a_{ij} \frac{\partial u}{\partial x_j}
$$

So the gradient of a function transforms as a Cartesian tensor of rank 1.
5) Tijkm is apparently 4a 5th rank tensor. \( S_{ijm} \) is a 2nd rank tensor. So the outer product would be a 7th rank tensor. Summing over \( l \) reduces that by 2 and summing over \( m \) reduces by another 2, so the outer product:

\[
T_{ijklm} \rightarrow \text{will be a 3rd rank tensor.}
\]

13) Start with \( T_{\beta\delta} \) and construct \( (T_{\beta\delta} + T_{\delta\beta}) \)
Now swap \( \beta, \delta : (T_{\delta\beta} + T_{\beta\delta}) \) which is the same
Next, construct \( (T_{\beta\delta} - T_{\delta\beta}) \) and swap \( \beta, \delta \)

\[
(T_{\delta\beta} - T_{\beta\delta}) = -(T_{\beta\delta} - T_{\delta\beta}) \text{: antisymmetric}
\]
\[ \frac{\mathbf{L}}{m} = \left[ \begin{array}{c} r^2 \mathbf{w}^2 - (xw_x + yw_y + zw_z)(x^2 + y^2 + z^2) \\
= (x^2 + y^2 + z^2)(w_x x + w_y y + w_z z) \\
- (xw_x + yw_y + zw_z)(x^2 + y^2 + z^2) \\
\end{array} \right] \\
= \mathbf{x} \left[ (x^2 + y^2 + z^2)w_x - (x^2w_x + y^2w_y + z^2w_z) \right] \\
+ \mathbf{y} \left[ (x^2 + y^2 + z^2)w_y - (x^2w_x + y^2w_y + z^2w_z) \right] \\
+ \mathbf{z} \left[ (x^2 + y^2 + z^2)w_z - (x^2w_x + y^2w_y + z^2w_z) \right] \\
= \mathbf{x} \left[ (y^2 + z^2)w_x - xyw_y - xz w_z \right] \\
+ \mathbf{y} \left[ -xyw_x + (x^2 + z^2)w_y - yz w_z \right] \\
+ \mathbf{z} \left[ -xz w_x - yz w_y + (x^2 + y^2)w_z \right] \\
= \begin{bmatrix} y^2 + z^2 & -xy & -xz \\
yx & (x^2 + y^2) & -yz \\
-zx & -zy & (x^2 + y^2) \end{bmatrix} \begin{bmatrix} w_x \\
w_y \\
w_z \end{bmatrix} \]
If I use the summation convention, does:

\[ \frac{L_i}{m} = x_i x_j \omega_i - x_i x_j \omega_j \]

give the same result?

How about:

\[ L_i = \int dm \left[ x_j x_j \omega_i - x_i x_j \omega_j \right] \]

\( \mathbf{2} \)

\[ I_{yx} = 1(-0(1)) + 2(-(-1)(1)) = 2 \quad \checkmark \]

\[ I_{yy} = 1(0^2+1^2) + 2(1^2+0^2) = 3 \quad \checkmark \]

\[ I_{yz} = 1(-1(1)) + 2(-(-1)(0)) = -1 \quad \checkmark = I_{zy} \]

\[ I_{zx} = 1(-1(0)) + 2(-(-1)(0)) = 0 = I_{xz} \]

\[ I_{zz} = 1(0^2+1^2) + 2(1^2+(-1)^2) = 5 \quad \checkmark \]
So \( I = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 5 \end{bmatrix} \)

I can diagonalize this matrix as in Ch 3:

\[
\begin{vmatrix} 4-\lambda & 2 & 0 \\ 2 & 3-\lambda & -1 \\ 0 & -1 & 5-\lambda \end{vmatrix} = 0
\]

\[
(4-\lambda)[(3-\lambda)(5-\lambda)-1]-2[2(5-\lambda)] = 0
\]

\[
(4-\lambda)(3-\lambda)(5-\lambda)-(4-\lambda)-4(5-\lambda) = 0
\]

\[
36 - 42\lambda + 12\lambda^2 - \lambda^3 = 0
\]

\[
(6-\lambda)(6-6\lambda+\lambda^2) = 0
\]

So the roots are \( \lambda = 6 \)

\[
2\lambda = \frac{6 \pm [36-24]^{1/2}}{2}
\]

\[
= 3 \pm \sqrt{3}
\]
We get e-vectors from

\[
\begin{bmatrix}
4 - \lambda & 2 & 0 \\
2 & 3 - \lambda & -1 \\
0 & -1 & 5 - \lambda
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = 0
\]

\[(4 - \lambda)x + 2y = 0\]
\[-y + (5 - \lambda)z = 0\]

For \( \lambda = 6 \):

\[-2x + 2y = 0 \quad \Rightarrow \quad y = x\]
\[-y - z = 0 \quad \Rightarrow \quad y = -z\]

So \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \) works

For \( \lambda = 3 \pm \sqrt{3} \):

\[(4 - 3 \pm \sqrt{3})x + 2y = 0\]

or \( y = \frac{-1 \pm \sqrt{3}}{2} x \)

\[ \Rightarrow y = (5 - \lambda)z = (5 - 3 \pm \sqrt{3})z\]

\[= (2 - \sqrt{3})z\]
2cut \, choose \, x = 1. \, then, \, for \, \lambda = 3 + \sqrt{3}
\begin{bmatrix}
1 \\
-1 + \sqrt{3} \\
-1 - \sqrt{3}
\end{bmatrix} = \begin{bmatrix}
\frac{2}{2(2 + \sqrt{3})} \\
1 + \sqrt{3} \\
2(2 + \sqrt{3})
\end{bmatrix}
\begin{bmatrix}
\frac{-2}{1 + \sqrt{3}} \\
1 \\
2 + \sqrt{3}
\end{bmatrix}
Is this the same as Dr. Beas' result?

All for \, \lambda = 3 - \sqrt{3}
\begin{bmatrix}
1 \\
-\sqrt{3} - 1 \\
-\sqrt{3} - 1
\end{bmatrix} = \begin{bmatrix}
\frac{-2}{2(2 + \sqrt{3})} \\
1 + \sqrt{3} \\
2 + \sqrt{3}
\end{bmatrix}
\begin{bmatrix}
\frac{-2}{1 + \sqrt{3}} \\
1 \\
2 + \sqrt{3}
\end{bmatrix}
Again, is that the same?
\[ I_{xx} = 1^2 + 1^2 + 1^2 + 1^2 = 4 \quad I_{yy} = -11 - (-1 \times 1) = 0 = I_{yx} \]

\[ I_{xx} = -1 \cdot 1 - (-1 \times 1) = 0 = I_{zx} \]

\[ I_{yy} = 1^2 + (-1)^2 + 1^2 = 4 \quad I_{yz} = -11 - (1 \cdot 1) = -2 = I_{zy} \]

\[ I_{zz} = (1^2 + 1^2 + (-1)^2 + 1^2 = 4 \]

\[ \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \]

The eigenvalues are from:

\[
\begin{vmatrix} 4 - \lambda & 0 & 0 \\ 0 & 4 - \lambda & -2 \\ 0 & -2 & 4 - \lambda \end{vmatrix} = (4-\lambda)[(4-\lambda)^2 - 4] = 0
\]

\[(4-\lambda)(16 - 8\lambda + \lambda^2 - 4) = 0\]

\[(4-\lambda)(12 - 8\lambda + \lambda^2) = 0\]

\[(4-\lambda)(6 - \lambda)(2 - \lambda) = 0\]
The e-vectors are from:

\[
\begin{bmatrix}
4-\lambda & 0 & 0 \\
0 & 4-\lambda & -2 \\
0 & -2 & 4-\lambda
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = 0
\]

So immediately \((4-\lambda)x = 0\) so if \(\lambda \neq 4\), \(x = 0\). If \(\lambda = 4\), then \(x\) is indeterminate. The other 2 expressions are:

\[(4-\lambda)y = 2z \quad 2y = (4-\lambda)z\]

So, if \(\lambda = 2\): \(y = 2, x = 0\)

if \(\lambda = 6\): \(y = -\frac{2}{3}, x = 0\)

if \(\lambda = 4\): \(y = \frac{2}{3}, x \neq 0\)
So the axis: \((0, 1, 1)\)

It goes from the origin through a point midway between the two masses: \((1, 1, 0)\). Each mass is 1 unit away from this axis. So the moment of inertia is: 
\[
I = (1)(1)^2 + (1)(1)^2 = 2
\]

The axis \((0, 1, 1)\) is perpendicular to the plane containing the origin and the two masses. The masses are \(\sqrt{3}\) units from this axis.

\[
I = (1)(\sqrt{3})^2 + (1)(\sqrt{3})^2 = 6
\]

Finally, the axis \((0, 1, 0)\) is the x-axis. Each mass is \(\sqrt{2}\) away, so

\[
I = (1)(\sqrt{2})^2 + (1)(\sqrt{2})^2 = 4
\]