Results of Example 5.11
\[ \vec{E}(r, \theta, \phi) = \begin{cases} \frac{\mu_0 R \omega}{3} & rsin\theta \hat{\phi} \quad r \leq R \\ \frac{\mu_0 R^4 \omega}{3} \frac{\sin \theta}{r^2} \hat{\phi} & r > R \end{cases} \]

Where \( R \) is the radius of the source of charge \( Q \), with charge density \( \rho = \frac{Q}{4\pi R^2} \). To use this in prob 5.30, I'll construct a spherical shell from the part of the solid sphere.
Look carefully at that figure.
There are a lot of lines. Redraw it yourself if you need too — it’s how I set up the integrals I’m about to do.
Let’s do the easy one first [how do I know which one is easy before I do them? — I’m a trick — I actually already did them].

To adapt the 5.11 results, identify the inner radius of the shell as \( r' \) (so \( R \to r' \)).
And that \( \sigma = \frac{dQ}{dA'} \) or \( P = \frac{dQ}{dr} \).

So that \( dQ = \sigma dA' = P dA' \)
and \( \sigma = P \frac{dr}{dA'} = \sigma \frac{4\pi r'^2 \, dr'}{4\pi r^2} = 0 \, dr' \)
Then, results of 5.11 become

\[
\frac{dA(r, \theta, \phi)}{dr} = \frac{\mu_0 r^4}{3} \frac{1}{r^3} \sin \theta \Phi \begin{cases} 
1 & r = r' \\
\frac{1}{r^2} \sin \theta \Phi & r > r'
\end{cases}
\]

Now we are ready to integrate. The easy one is when the field point (at \( r \)) is outside the solid sphere. Then \( r > r' \) always and the integral is:

\[
A(r) = \frac{\mu_0 \omega \Phi}{3} \frac{1}{r^2} \Phi \int_0^r r'^4 \, dr' = \frac{\mu_0 \omega \Phi}{3} \frac{\sin \theta \Phi}{r^2} \frac{R^5}{5}
\]
It's more complicated if the field point is inside the sphere \((r < R)\) because there are two possibilities: the source is inside the field point \((r' < r)\) or outside \((r' > r)\) and I have different expression for \(\overrightarrow{dA}'\) so that looks like:

\[
\overrightarrow{dA}' = \frac{m \omega p}{3} \frac{\sin \theta'}{r'^2} r'^4 \, dr' \quad r' < r
\]

\[
\frac{m \omega p \sin \theta'}{3} r \cdot r' \, dr' \quad r' > r
\]

so the integral becomes:

\[
\overline{A} = \frac{m \omega p \sin \theta'}{3} \left\{ \int_0^r \frac{r'}{r^2} \overrightarrow{r'} \, dr' + r \int_r^R \overrightarrow{r'} \, dr' \right\}
\]
Note that up to now, you (the human being) has to do this. Now you can let a computer do the integrals.

\[
\frac{\Delta}{A} = \frac{\pi \rho \sin \theta}{3} \left\{ \frac{r^3}{5} + \frac{1}{2} r (R^2-r^2) \right\}
\]

\[
= \frac{\pi \rho \sin \theta}{3} \frac{1}{2} r (R^2 - \frac{3}{5} r^2)
\]

Is \( \Delta \) continuous at \( r = R \)?

(Does it have to be?)

(What does this say about \( R \) ? Eq 5.77, 78)

Rewrite the potential in terms of the charge, \( Q \), of the solid sphere.
To find the field, we take the curl:

\[ \mathbf{B} = \frac{\mathbf{d} \times \mathbf{A}}{4\pi} \] is inside.

\[ \frac{1}{r} \mathbf{e}_\theta \left[ \frac{1}{r} \mathbf{e}_\theta \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{e}_\theta \mathbf{e}_\theta \right] \]

\[ \mathbf{A} = \frac{M_0 \omega_0}{4\pi} \frac{r}{2} \sin \theta \left( \frac{1}{r} \left( 1 - \frac{3}{5} \frac{r^2}{R^2} \right) \right) \hat{\mathbf{r}} \]

\[ = \frac{M_0 \omega_0}{4\pi} \frac{r}{2} \left\{ \frac{1}{r} \sin \theta \left( 2 \mathbf{e}_\theta - \frac{3}{5} \frac{r^2}{R^2} \right) \hat{\mathbf{r}} \right\} \]

\[ + \left[ \frac{2 \sin \theta}{R} \left( \frac{6}{5} \frac{r^2}{R^2} - 1 \right) \right] \hat{\theta} \]
5.30. Outside \( \vec{A} = \frac{\mu_0}{4\pi} \frac{w Q R^2}{r^2} \ \sin \theta \ \hat{\phi} \)

So \( \vec{B} = \frac{\mu_0}{4\pi} \frac{w Q R^2}{r^2} \left\{ \left[ \frac{1}{r \sin \theta} \partial_r \left( \frac{\sin^2 \theta}{r} \right) \right] \hat{r} + \left[ -\frac{1}{r} \partial_r \left( \frac{\sin \theta}{r} \right) \right] \hat{\theta} \right\} \)

\[ = \frac{\mu_0}{4\pi} \frac{w Q R^2}{r^2} \left\{ \frac{2 \cos \theta}{r^3} \hat{r} + \frac{\sin \theta}{r^3} \hat{\theta} \right\} \]

**Consider** \( \vec{A} \) outside \( \vec{B} \). Can you identify the magnetic dipole using eqns 5.87 \& 5.88?

If a proton is a solid sphere, use the dipole moment \& angular momentum to find the speed of the equator. Use Eqn 5.86 to get \( m \). SAME?