the electric field of a line of charge is
\[ \mathbf{E} = \frac{2\lambda}{4\pi\varepsilon_0 l} \]

the force per unit length on the other line is
\[ \frac{dF}{dl} = \left( \frac{dq}{dl} \right) \mathbf{E} = \lambda \cdot \frac{2\lambda}{4\pi\varepsilon_0 d} = \frac{\lambda^2}{2\pi\varepsilon_0 d} \]

magnetic force per unit length on two currents
\[ \frac{dF}{dl} = \frac{\mu_0 I^2}{2\pi d} (\lambda V)(\lambda V) = \frac{\mu_0 I^2 V^2}{2\pi d} \]

the ratio of magnetic to electric force is:
\[ \frac{dF_{\text{mag}}}{dF_{\text{elec}}} = \frac{\mu_0 I^2 V^2}{2\pi d} = \frac{\mu_0 \varepsilon_0 V^2}{2\pi d} = \frac{1}{2} \frac{2\pi d}{\varepsilon_0} = \frac{1}{2} \frac{2\pi d}{\varepsilon_0} = \frac{1}{2} \frac{2\pi d}{\varepsilon_0} = \frac{1}{2} \frac{2\pi d}{\varepsilon_0} \]

where \( V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \) is apparently a speed. Is it reasonable?
From prob 5.5 - this is a surface current of \( \frac{1}{2} = \frac{2}{2\pi a} \).

Because of the symmetry, \( \vec{B} = B\hat{\phi} \).

Choose a circle of radius \( s \) to calculate Ampere's Law.

Along the circle \( dl = s \, d\phi \).

So \( \oint B \cdot dl = B(s)\hat{\phi} \cdot s \, d\phi \).

\[ \oint B \cdot dl = B(s) \cdot s \int d\phi = B(s) \cdot 2\pi s \]

Circle

If the circle is within the wire so that \( I_{\text{enclosed}} = 0 \), we have \( B(s) \cdot 2\pi s = \mu_0 I_{\text{enclosed}} = 0 \).

So \( B(s) = 0 \) inside \( (s < a) \).
if the circle is outside, then \( I_{\text{enclosed}} = I \) (all of the current).

\[
B(s) \cdot 2\pi s = \mu_0 I \quad n
\]

\[
\vec{B}(s) = \frac{\mu_0 I}{2\pi s} \hat{n}
\]

Does the magnetic field satisfy the BC's in 5.4.2?

b) From Example 5.4, \( k = \frac{3}{2\pi} \frac{I}{a^3} \)

\[
\vec{J} = \frac{3}{2\pi} \frac{I}{a^2} \left( \frac{s}{a} \right) \hat{z}
\]

So

As before \( B(s) = B(s) \cdot 2\pi s = \mu_0 I_{\text{enclosed}} \)

\[
I_{\text{enclosed}} = \int \vec{J} \cdot d\vec{A} = \int \frac{3}{2\pi a^3} \frac{I}{s} \hat{z} \cdot ds \cdot s \cdot d\phi \cdot \hat{z}
\]

Surface
5.14b \( \text{cm}^4 \) = \( \frac{3}{2\pi} \frac{I}{a^3} \int s^2 ds \ d\phi \)

\[
= \frac{3I}{a^3} \int s^2 ds
\]

- If Inside \( s < a \)

\[
= \frac{3I}{a^3} \int_0^s s^2 ds = \frac{I}{a^3} s^3 = I \left( \frac{s}{a} \right)^3
\]

- If Outside \( s > a \)

\[
= \frac{3I}{a^3} \int_a^s s^2 ds = \frac{I}{a^3} a^3 = I
\]

So Inside \( B(2\pi s) = \mu_0 \left( \frac{s}{a} \right)^3 \)

\[
\frac{1}{B} = \frac{\mu_0}{2\pi a} \left( \frac{s}{a} \right)^2 \phi
\]
And outside
\[ B(2\pi s) = \mu_0 I_0 \]

\[ \hat{B} = \frac{\mu_0 I_0}{2\pi s} \hat{\phi} \]

\( \checkmark \) Is the \( \hat{B} \) satisfy BC's surface?

\( \checkmark \) Use \( \frac{1}{s} \) from Prob 5.5b) a\&l

find \( \hat{B} \)
Choose a point $z < -a$ (below the current).

From the Biot-Savart Law, $\mathbf{B}$ will have component along $z$ and along $y$. Depending upon where $J$ is, the $z$ component can be either $+$ or $-$, but the $y$ component must be $(+)$.

So because the current is infinite, the $z$ components will cancel, so $\mathbf{B} = B(z) \hat{y}$.

$\square$ Make the same argument if $-a < z < 0$, $0 < z < a$, and $a < z$. 
So below x-y plane $\vec{B} = B \hat{y}$

d & bore $\vec{B} = B (-\hat{y})$

Use this symmetry to use Ampère's Law:

Along $1 \parallel 3, \int \vec{B} \cdot d\ell = 0$

Along $1 \parallel 4, \int \vec{B} \cdot d\ell = Bky$

So $\int \vec{B} \cdot d\ell = \int_{2} B \, dy = 2Bh$

Since $J$ is uniform; $\int J \cdot dA = JA = J(4)(2a)$

So $2Bh = \mu_o h J 2a$, so $B = \mu_o J / 2a$

a. $\vec{B} = \frac{1}{2} \mu_o (J 2a) \hat{y}$ for $z < -a$

b. $\frac{1}{2} \mu_o (J 2a) (-\hat{y})$ for $z > a$

(Compare Eqn 5.58)
Now use a path partly inside the slab:

Again \( 1 \leq 3 \) give \( \overrightarrow{B} \cdot \overrightarrow{dA} = 0 \)

Along 2: \( \int \overrightarrow{B} \cdot d\overrightarrow{A} = B_h = \mu_0 J_h h \)

Along 1: \( \int \overrightarrow{B} \cdot d\overrightarrow{A} = B_l = B_{1l} h \) (looking for \( B \) inside)

\( \overrightarrow{B} \cdot d\overrightarrow{A} = (B_{1l} + \mu_0 J_a) h \)

I enclosed = \( J \cdot h \cdot (a - z) \)

\( (B_{1l} + \mu_0 J_a) h = \mu_0 J_h (a - z) \)

\( B_{1l} = -\mu_0 J \frac{z}{a} \) or \( \overrightarrow{B}_{1l} = \mu_0 J \frac{z}{a} (-\hat{y}) \)

\( 0 < z < a \)

Show that \( \overrightarrow{B}_{1l} = \mu_0 J \frac{z}{a} \hat{y}, -a < z < 0 \)
5.15 cm

So finally,

\[ \frac{1}{B} = \begin{cases} 
  M_0Jz (-\hat{y}) & a < z \\
  M_0Jz (-\hat{y}) & 0 < z < a \\
  M_0Jz (+\hat{y}) & -a < z < 0 \\
  M_0Jz (+\hat{y}) & z < -a 
\end{cases} \]
\textbf{5.17}\quad \textbf{GRAPH}\quad \textbf{SM}\quad \textbf{4TH}\quad \textbf{HW}

1) \quad \frac{+\gamma}{xk} \to \frac{0}{0^+} \to \frac{0}{0^-} \to \frac{1}{1} = \frac{0}{0^-} \to \frac{0}{0^-} \to \frac{-\gamma}{xk} \to \frac{0}{0^-} \to \frac{1}{1} = \frac{0}{0^-} \to \frac{0}{0^-} (-\gamma)

From Example 5.8 -

In region I, the \(+\gamma\) plate creates a field \(B = \frac{1}{2} \mu_0 k\) to the \textbf{left}

and the \(-\gamma\) plate creates a field \(B = \frac{1}{2} \mu_0 k\) to the \textbf{right}

So, In region I \(\mathbf{\overrightarrow{B}} = 0\)

In region II, Both create a field \(\frac{1}{3} \mu_0 k\) to the \textbf{right}, so \(\mathbf{\overrightarrow{B}} = \mu_0 k\) \textbf{right}

\(\oplus\) Argue that in region III, \(\mathbf{\overrightarrow{B}} = 0\)

\(\oplus\) Does this satisfy 5.4.2 Eqn. 5.74 \& 75
1) Just below the upper plate, the field is $\vec{B} \approx \mu_0 K \hat{r}$ right and just above $\vec{B} = 0$. (Prob 5.15 suggests $\vec{B}$ would decrease linearly from $\mu_0 K$ to zero.)

So the average value of $\vec{B}$ is $\frac{1}{2} \mu_0 K$ (right).

The current by a small area of charge would be $I d\vec{e} = \frac{d\vec{q}}{dt} = d\vec{q} \cdot \vec{v} = \sigma \vec{v} dA = K dA$

So the force is $d\vec{F} = I d\vec{e} \times \vec{B}_{av}$

$= K dA \times \vec{B}_{av}$

And the pressure is

$\frac{d\vec{F}}{dA} = K \times \vec{B}_{av} = K \frac{1}{2} \mu_0 K (\hat{u} \times \hat{r})$

$\vec{P} = \frac{1}{2} \mu_0 K^2 (\hat{u} \hat{p}) = \frac{1}{2} \mu_0 (\sigma \vec{v})^2 \hat{u} \hat{p}$
5.17c) Recall that the pressure from the electrical pressure is $p = \frac{\sigma^2}{2\varepsilon_0}$ Eq. 2.51 (attractive in this case)

For the pressures to balance:

$$\frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0}{2} k^2 = \frac{\mu_0}{2} (\sigma V)^2$$

or $\sigma^2 = \frac{1}{\mu_0 \varepsilon_0}$ (Seems familiar)

5.19 According to Stokes's Theorem, you can use ANY surface bounded by the Amperean loop, and the answer has to be the same. If you allow the magnetic field to CHANGE (not MAGNETOSTATICS you don't always get the same answer. This is how MAXWELL fixed Ampere's Law and discovered EM radiation.
a) \[ p = \frac{9 \text{ mol}}{\text{Vol}} = \frac{9 \text{ mol} \cdot \text{NA}}{\text{Vol}} = \frac{9 \text{NA}}{\text{Vol}} \cdot \frac{\text{mass}}{\text{Mol. wght}} \]

\[ = \frac{9 \text{NA}}{M_w(\text{mass})} \]

\[ = \frac{(1.6 \cdot 10^{-19}) \cdot (6.0 \cdot 10^{23}/\text{mol}) \cdot (9.8/\text{mL})}{63.5 \text{ g/mol}} \]

\[ = 1.3 \cdot 606 \text{ C/mL} \cdot \frac{10^6 \text{ mL}}{\text{m}^3} = 1.3 \cdot 10^9 \text{ C/m}^3 \]

b) \[ J = \rho \nu \Rightarrow \nu = \frac{J}{\rho} = \frac{J}{AP} = \frac{I}{\pi R^2 \rho} \]

\[ = \frac{1 \text{ A} \cdot \text{cm}}{\pi (0.5 \cdot 10^{-3} \text{m})^2 (13.6 \cdot 10^9 \text{C/m})} = 9.1 \cdot 10^{-5} \text{ m/s} \]

\[ \approx 0.34 \text{ m/hr} \approx 2 \text{ mi/year} \]

I don't need the electrons by me to go there - I need the electrons already there to accelerate...
\[ F = ILB = IL \left( \frac{\mu_0 I}{2\pi s} \right) \]

So \[ \frac{F}{L} = \frac{\mu_0 I^2}{2\pi s} = \frac{4\pi \times 10^7 N/A^2 \times \frac{1A^2}{10^{-2}m}}{2\pi} = 2 \times 10^3 N/m \]

d) \[ \lambda = \rho A \quad \text{and} \quad I = \lambda U \]

So I can write \[ \frac{E}{L} = \frac{\lambda U}{2\pi s} \]

Hence, the compensating charge, I have an electric force per unit length \[ \frac{dF}{dL} = \frac{dq \cdot E}{dL} = \lambda E = \lambda \frac{\lambda U^2}{2\pi \rho_0 s} = \frac{\lambda^2 U^2}{2\pi \rho_0 s} \]

So the ratio of
\[ \text{Magnetic} = \frac{\mu_0 \lambda^2 U^2}{2\pi \rho_0 s} \]
\[ \text{Electric} = \frac{\lambda^2}{2\pi \rho_0 s} \]
\[ = \mu_0 \varepsilon_0 U^2 = \left[ \frac{U}{\sqrt{\mu_0 \varepsilon_0}} \right]^2 \]

*Put in the numbers

*Get used to that ratio - it happens a lot*
Amperes Law says that:
\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]

\[ \nabla \cdot \vec{B} = 0 \]

Amperes Law claims that:
\[ \nabla \cdot \vec{J} = 0 \]

But we know that:
\[ \nabla \cdot \vec{J} = -\frac{\partial \vec{E}}{\partial t} \quad \text{(Eq 5.29)} \]

So apparently Amperes Law only works when \( \rho \) is unchanging.

Maxwell came up with the fix.

(Using Eq 5.29 and Gauss Law \( \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \))

Faraadays Law is fine (as long as there are no magnetic monopoles)