1) In class, we showed that the electric field a distance $x$ away from a finite line of charge is:

$$E = \frac{2k\lambda}{x} \frac{L/2}{\sqrt{x^2 + (L/2)^2}}$$

where the symbols are as usually defined. Consider four such "sticks" of charge arranged in a square and find the electric field at the center of the square.

The center of the square will be along the axis of each stick, so the equation applies. It will be equal distance from each stick, so the magnitude of $E$ produced by each stick will be the same. And since sticks on opposite sides produce a field in opposite direction, the field produced will cancel in pairs.

So

$$E = 0$$
Imagine a stick of charge of length L, carrying charge Q, oriented along the y-axis and centered at the origin (i.e., the same as in problem 1). If you did as I asked, you calculated the potential a distance x away and got: \[ V(x) = k \lambda \ln \left[ \frac{\sqrt{1 + (L/2x)^2} + L/2x}{\sqrt{1 + (L/2x)^2} - L/2x} \right] \].

A) What form do you expect this equation to take in the limit \( x \to \infty \)?

Should look like a point charge:

\[ V(x) \approx \frac{k \lambda Q}{x} \]

B) SHOW that you get this form by letting x become large compared to L in the above equation. (HINT: You'll need \( \sqrt{1+u} \approx 1 + \frac{1}{2} u \) and \( \ln[1+u] \approx u \) for u small.) (OTHER HINT: You only need to keep terms like L/2x in your approximations.)

\[
\begin{align*}
k \lambda \ln \left[ \frac{\sqrt{1 + (L/2x)^2} + L/2x}{\sqrt{1 + (L/2x)^2} - L/2x} \right] & \approx k \lambda \ln \left[ \frac{1 + \frac{1}{2} (L^2x^2) + \frac{L}{2x}}{1 + \frac{1}{2} (L^2x^2) - \frac{L}{2x}} \right] \\
& \approx k \lambda \left\{ \ln \left[ 1 + \frac{L}{2x} \right] - \ln \left[ 1 - \frac{L}{2x} \right] \right\} \\
& \approx k \lambda \left\{ \frac{L}{2x} - (-\frac{L}{2x}) \right\} = \frac{k \lambda L}{x} \\
& = \frac{k \lambda Q}{x} \quad \checkmark \quad \text{QED}
\end{align*}
\]
3) Consider two spherical conductors (label them A and B), with different radii \( R_A < R_B \). Initially, each carries the same charge \( Q \). Assume they are then connected with a wire and allowed to come to equilibrium. Which sphere ends with more charge? (Explain your answer – a wild guess gets zero credit). (HINT: Once they are connected with a wire, they become one conductor – what do you know about the potential at the surface of a conductor?) (OTHER HINT: What is the voltage at the surface of a charged spherical conductor?)

A conductor at equilibrium has its surface at constant potential. So, initially the voltages are different: \( V_A = \frac{Q}{R_A} > V_B = \frac{Q}{R_B} \)

So charges will move around until both spheres (AND the wire) are at the same potential: \( V_A = \frac{kQ_A}{R_A} = V_B = \frac{kQ_B}{R_B} \)

Since \( R_A < R_B \), \( Q_A < Q_B \)

So B winds up with more charge.
4) In class, we discussed a monopole (the technical name for a point charge) and a dipole (two point charges of opposite sign near one another). You would think the next in line would be a tripole, but it isn’t—it’s a quadropole. There are various ways to set up a quadropole and one way is two oppositely directed dipoles near one another. For example, a charge +q at the point (x,y)=(a,a); another +q charge at (-a,-a); a charge -q at (a,-a) and another -q charge at (-a,a). (Ask me and I’ll sketch it on the board).

A) Show that the voltage due to this charge distribution is zero everywhere along the x and y axes.

For a point charge, \( V = \frac{kq}{r} \). So if you’re equal distance from two opposite charges (\( V = 0 \)). Along y-axis, you’re equally distant from 2 upper \( \frac{3}{2} \) lower charges (\( V = 0 \)).

Along x-axis, you’re equal distance from 2 left \( \frac{1}{2} \) right charges. So \( V_{\text{total}} = 0 \)

B) Does this mean that the electric field is zero everywhere along the x and y axes? (HINT: No it doesn’t—explain why.)

No. \( E = \frac{dV}{dr} \), so all we know is that along y-axis, \( E_y = -\frac{dV}{dy} = 0 \) and along x-axis, \( E_x = -\frac{dV}{dx} = 0 \).

C) If possible, indicate the direction of the electric field along the four axes (+ and - x axis and + and - y axis). If it’s not possible, explain what you would need to do so.

Along +y, upper dipole makes \( \vec{E} \) in (-\( \hat{y} \))

all lower dipole makes \( \vec{E} \) in (+\( \hat{y} \)). But upper dipole is closer, \( E \) so \( \vec{E}_{\text{tot}} \) is (-\( \hat{y} \)). Similarly, along -y, \( \vec{E} \) is (+\( \hat{y} \)); along +x, \( \vec{E} \) is (-\( \hat{x} \))

\( \vec{E} \) along -x, \( \vec{E} \) is (+\( \hat{x} \))