P22] a) \( I = nAuE = nAuE \)

\[ nAu: \left( \frac{\text{m}^3}{\text{m}^2 \cdot \text{s}} \right) = \frac{\#}{\text{s}} - \text{number per time} \]

\[ nAuE: \left( \frac{\text{m}^3}{\text{m}^2 \cdot \text{s}} \right) \left( \frac{\text{m}}{\text{s}} \right) = \frac{\#}{\text{s}} \]

b) \( n = \text{number per unit volume of mobile charges} - \left[ \text{m}^{-3} \right] \)

c) \( A = \text{cross sectional area of conducting path} - \left[ \text{m}^2 \right] \)

d) \( \bar{v} = \text{drift speed of mobile charges} - \left[ \text{m/s} \right] \)

e) \( u = \text{mobility} - \left[ \frac{\text{m}^2}{\text{V} \cdot \text{s}} \right] \)
$N = \# \text{ of donor molecules}$

$\Delta t = \text{"life" of battery}$

$I = \text{current} = \frac{\# \text{electrons} \times 9}{t}$

$\Rightarrow I \Delta t \leq Ne$

$\Delta t \leq \frac{Ne}{I} = \frac{(0.5)(6 \cdot 10^{23})(1.6 \cdot 10^{-19} e)}{0.3 \text{ A/s}}$

$= 16 \cdot 10^4 \text{s} \approx 44 \text{ hours}$

( google "allain battery" )
a) The compass needle is a magnetic dipole, so it will feel a torque that tends to align it with the field. The Earth's $\vec{B}$ will rotate it to point North (to the right). Since it points a few degrees above that (about NW), the net $\frac{1}{2} \vec{B}$ points NW. So the wire creates a field up (or W) above the wire. So the conventional current moves left, so electrons are moving right.
P34 (cont.)

b) Since the bulbs are identical, intuition suggests the electron current divides equally between the two branches (in CH 19, we'll see how to deal with this properly). So \(1.5 \times 10^8\) pass \(P_2\).

c) Since the bulbs are identical, the bright ren of \(B_2\) is \(B_3\) should be the same, i.e. \(B_1\) should be brighter (not necessarily \(2x\) though).
\[ P^{34}(cm^3/d) \] \[ i = nAuE \]

\[ E = \frac{i}{nAu} = \frac{3 \times 10^{18} \text{V}}{(6.3 \times 10^{25} \text{m}^3)(10^{-8} \text{m}^2)(1.2 \times 10^{-4} \text{m}^2/\text{V.s})} \]

\[ = 40 \frac{\text{V}}{\text{m}} \]

Since the electrons are drifting to the right, the electric force is to the right, so the electric field is to the left.
I'm guessing this is a C battery
[looked it up - close, but not exact]
[Radius is more like 1.3 cm
www.powersstream.com]
[and it's a lithium battery]

So the voltage difference between the ends is 1.7V

Voltage difference between two oppositely charged plates (along the axis) is:

$$\Delta V = -\int_E \cdot dl = +\int [E + E_z] \, dx$$

-\(Q\)  \hspace{1cm} +\(Q\)  \hspace{1cm} \(x\)
\( E_- = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{x}{\sqrt{L^2 + x^2}} \right] \)

\( E_+ = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{(L-x)}{\sqrt{R^2 + (L-x)^2}} \right] \)

There are four integrals to do:

\[
\int_{0}^{L} \frac{\sigma}{2\varepsilon_0} \cdot dx = \frac{\sigma}{2\varepsilon_0} L \quad \text{(twice)}
\]

\[-\int_{0}^{L} \frac{\sigma}{2\varepsilon_0} \frac{x}{\sqrt{R^2 + x^2}} \, dx = -\frac{\sigma}{2\varepsilon_0} \int_{0}^{L} \frac{x \, dx}{\sqrt{R^2 + x^2}}
\]

Substitute \( u = R^2 + x^2 \):

\[
-\frac{\sigma}{2\varepsilon_0} \int_{0}^{L} \frac{1}{2} u^{1/2} \, du = -\frac{\sigma}{2\varepsilon_0} \left[ \frac{u^{3/2}}{3/2} \right]_{R^2}^{R^2 + L^2}
\]

\[
= -\frac{\sigma}{2\varepsilon_0} \left( \sqrt{R^2 + L^2} - R \right)
\]
\[ P_{36cm^4} \]

Do the last one. You'll see it's the same result:

\[ -L \int_{0}^{L} \frac{(L-x)dx}{\sqrt{x^2 + (L-x)^2}} = -\frac{\sigma}{2\varepsilon_0} \left( \sqrt{R^2 + L^2} - R \right) \]

So \( \Delta V = \frac{\sigma}{\varepsilon_0} \left\{ L + R - \sqrt{R^2 + L^2} \right\} \)

Do it with the approximations:

1) \( E \approx \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{R}{L} \right) \)

2) \( E \approx \frac{\sigma}{2\varepsilon_0} \)

How big is the error?
Problem 18

18. (a) 

\[ \varepsilon_0 \frac{\Delta V}{(L+R)[1 - \sqrt{\frac{L-R}{L+R}}]} = \varepsilon_0 \] 

\[ \frac{\Delta V}{(5.5 \text{ cm})[1 - \sqrt{\frac{3.5}{5.5}}]} = 1.53 \frac{V}{\text{cm}} \] 

\[ \sigma = \varepsilon_0 (1.53 \frac{V}{\text{cm}}) = \left( \frac{1.53 \times 10^{-2} \text{ N m}^{-2}}{8.85 \times 10^{-12} \text{ C}^2 \text{ m}^{-2}} \right) \] 

\[ = 1.35 \times 10^{-9} \frac{\text{C}}{\text{m}^2} \]

\[ Q = \sigma A = (1.35 \times 10^{-9} \frac{\text{C}}{\text{m}^2})(\pi)(10^{-2} \text{ m})^2 \]

\[ = 0.42 \times 10^{-12} \text{ C} = 0.42 \text{ pC} \]

(b) It's no sufficient to repel much of any thing.
The voltage (relative to "infinity") of each sphere is:

\[ V_{\text{Large}} = \frac{Q}{4\pi\varepsilon_0 R} \quad V_{\text{Small}} = \frac{-9}{4\pi\varepsilon_0 R} \]

so the voltage difference is:

\[ \Delta V = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{R} + \frac{9}{r} \right) > 0 \]

(i.e., large sphere is at a larger potential, so (+) charges would flow from large to small, or (-) charges would flow from small to large, or both).

What would be different if the small sphere had (+) charge? Both had same charge?
The conventional current will be:

\[ I = e n A \overline{u} \]

\[ = e n A u E \]

\[ = e n A u \frac{\Delta V}{L} \]

\[ = \frac{e n A u}{4 \pi \varepsilon_0 L} \left( \frac{Q}{R} + \frac{q}{r} \right) \]

The electron current is

\[ i = \frac{I}{e} \text{ all the # of electrons} \]

\[ \Rightarrow N = i \Delta t = \frac{n A u}{4 \pi \varepsilon_0 L} \left( \frac{Q}{R} + \frac{q}{r} \right) \]

Where, over a short time, I assume that \( Q, q \approx \text{constant} \)

( don't decrease much )