p48] I will point the pen toward the foil. You should try it the other way: That is me

\[ E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{x(x+2)} \]

From prob P29: This field will polarize the foil.

Since the foil is metal, it will polarize until the field inside the metal foil is zero. The field produced by the charge on the surfaces of the foil will be:

\[ E = \frac{\sigma}{\varepsilon_0} \]

And so

\[ \frac{\sigma}{\varepsilon_0} = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{x(x+2)} \]
So the charge on each face of the foil will satisfy:

\[ \frac{q}{\varepsilon_0 \pi R^2} = \frac{1}{4 \pi \varepsilon_0} \cdot \frac{\varphi}{x(x + L)} \]

\[ q = \frac{R^2}{x(x + L)} \varphi \]

If I now treat the foil as a dipole, I can calculate the force exerted on the foil:

\[ \mathbf{F} = \mathbf{F}_+ + \mathbf{F}_- = qE(x + \frac{d}{2})\hat{\mathbf{y}} + qE(x - \frac{d}{2})\hat{\mathbf{y}} \]

\[ = q \left( \frac{1}{4 \pi \varepsilon_0} \frac{\varphi}{(x + \frac{d}{2})(x + \frac{d}{2} + L)} - \frac{1}{4 \pi \varepsilon_0} \frac{\varphi}{(x - \frac{d}{2})(x - \frac{d}{2} + L)} \right) \hat{\mathbf{y}} \]
I'll assume that \( d \) (the thickness of the foil) is much smaller than either the distance \( x \) or the length of the pen.

\[
\frac{F}{2} = \frac{qQ}{4\pi \varepsilon_0 x(x + L)} \left\{ \frac{1}{(1 + \frac{d}{2x})(1 + \frac{d}{2(x + L)})} - \frac{1}{(1 - \frac{d}{2x})(1 - \frac{d}{2(x + L)})} \right\}
\]

\[
= \frac{qQ}{4\pi \varepsilon_0 x(x + L)} \left\{ (1 + \frac{d}{2x})^{-1}(1 + \frac{d}{2(x + L)})^{-1} - (1 - \frac{d}{2x})^{-1}(1 - \frac{d}{2(x + L)})^{-1} \right\}
\]

\[
= \frac{qQ}{4\pi \varepsilon_0 x(x + L)} \left\{ (1 - \frac{d}{2x} - \frac{d}{2(x + L)}) - (1 + \frac{d}{2x} + \frac{d}{2(x + L)}) \right\}
\]

\[
= \frac{qQ}{4\pi \varepsilon_0 x(x + L)} \left\{ -\frac{d}{x} - \frac{d}{1(x + L)} \right\} \text{ (down)}
\]
\[ F = \frac{-Qq d}{4 \pi \varepsilon_0 x (x+L)} \left\{ \frac{L+2x}{x(x+L)} \right\} \text{(down)} \]

So the force is up (right?), as expected (right?).

From the polarizeability of the foil,

\[ q = \frac{R^2}{x(x+L)} \]

So:

\[ F = \frac{-Qd}{4 \pi \varepsilon_0 x(x+L)} \frac{R^2}{x(x+L)} Q \left\{ \frac{L+2x}{x(x+L)} \right\} \text{(up)} \]

\[ = \frac{Q^2}{4 \pi \varepsilon_0} \frac{d R^2}{[x(x+L)]^3} \text{(up)} \]

To lift the foil, this force must equal or exceed the weight:

\[ mg = \rho V g = \pi R^2 d \rho g \]
\[ \pi R^2 x \rho g = \frac{Q^2}{4 \pi \varepsilon_0 \left[ x(x+L) \right]^{3}} (L+2x) \]

\[ \frac{\left[ x(x+L) \right]^{3}}{L+2x} = \frac{Q^2}{4 \pi^2 \varepsilon_0 \rho g} \]

If I now assume (just see the ballpark that \( L \gg x \):

\[ x \approx \frac{Q^2}{4 \pi^2 \varepsilon_0 \rho g L^2} \]

\[ 1 \text{ m}^3 = \frac{Q^2}{\text{m}^2 \cdot \text{kg} \cdot \text{s}^2 \cdot \text{m}^3 \cdot \text{kg}^{-1}} = \text{m}^3 \]

So pick some numbers:

\[ Q \approx 50 \text{nC} \] (for no good reason - Example p602)

\[ L \approx 15 \text{cm} \]

\[ \rho \approx 2700 \text{ kg/m}^3 \]
\[ \text{P} = 18 \text{cm}^3 \]

\[ x^3 = \frac{(50 \times 10^{-9} \text{m})^2}{4 \pi^2 (8.85 \times 10^{-12} \text{C}^2 \text{m}^{-2}) (2700 \frac{\text{kg}}{\text{m}^3}) (10 \frac{\text{m}}{\text{s}})} \]

\[ = 1.2 \times 10^{-8} \text{m}^3 \]

\[ x \approx 0.0023 \text{m} \approx 2 \text{mm} \]

It jumped up a more like a cm. So what could change in the equation above to get closer? I measured the length as \( L = 15 \text{cm} \) (same as my guess above—who knows?)
P54 a) The sphere contributes no field inside, so the only field is due to the plates:

\[ E = \frac{2Q/A}{2\varepsilon_0} \left( \hat{u}_1 \right) + \frac{Q/A}{2\varepsilon_0} \left( \hat{u}_4 \right) = \frac{3Q/A}{2\varepsilon_0} \left( \hat{u}_4 \right) \]

\[ = \frac{3Q}{2\varepsilon_0 \frac{1}{R^2}} \left( \hat{u}_4 \right) \]

And this is the same through the region between the plates, so I'll just plug it in below.

b) To the field above, there is a field (also to the left) from the sphere, of magnitude

\[ E = \frac{9}{4\pi\varepsilon_0 (3r)^2} = \frac{9}{36\pi\varepsilon_0 r^2} \]

So the net field is:

\[ E = \frac{1}{4\pi\varepsilon_0} \left( \frac{6Q}{R^2} + \frac{9}{9r^2} \right) \left( \hat{u}_4 \right) \]
c) Now, the sphere creates a field of magnitude: \[ \frac{1}{4\pi\varepsilon_0} \frac{9}{18r^2} \]

with a vertical component of:

\[ \frac{1}{4\pi\varepsilon_0} \frac{9}{18r^2} \frac{1}{\sqrt{2}} \]

and a (left directed) horizontal component of:

\[ \frac{1}{4\pi\varepsilon_0} \frac{9}{18r^2} \frac{1}{\sqrt{2}} \]

So the net field is:

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \left( \frac{6\Phi}{R^2} + \frac{9}{18\sqrt{2}r^2} \right) \hat{r} + \frac{1}{4\pi\varepsilon_0} \left( \frac{9}{18\sqrt{2}r^2} \right) \hat{\Theta} \]
If the proton moved a distance, $s$, from the center of the electron "fog", it is at a place where the electron creates a field of magnitude:

$$E = \frac{e}{4\pi \varepsilon_0 R^3}$$

directed toward the origin, so there will be a restoring force of magnitude:

$$F = \frac{e^2}{4\pi \varepsilon_0 R^3}$$

to balance the applied force the external field exerts on the proton.

From above:

$$p = er = (4\pi \varepsilon_0 R^3) E$$
so the polarizability is clearly:

\[ \alpha = 4\pi \varepsilon_0 R^3 \]

5) \( \alpha \approx 4\pi \left( 8.85 \times 10^{-12} \text{ Nm}^2 \text{C}^{-2} \right) \left( 10^{-10} \text{ m} \right)^3 \)

\[ = 1.1 \times 10^{-40} \frac{(\text{C}^2)}{\text{kg}} \text{ (vs 0.7} \times 10^{-40} \frac{\text{C}^2}{\text{kg}} \text{) } \]

\[ c) \ \rho = \frac{\alpha E}{e} = \frac{\left( 1.1 \times 10^{-40} \frac{\text{C}^2}{\text{kg}} \right) \left( 10^6 \text{ N/m} \right)}{1.6 \times 10^{-19} \text{ C}} \]

\[ = 7 \times 10^{-16} \text{ m (roughly the size of the proton)} \]

d) \( F = \frac{e^2}{\alpha} r = k_s r \)

\[ \alpha k_s = \frac{e^2}{\alpha} = \frac{(1.6 \times 10^{-19}\text{C})^2}{0.74 \times 10^{-10} \frac{\text{C}^2}{\text{kg}}} = 346 \text{ N/m} \]