1) \textbf{(15 points)} Show \textit{quantitatively} that a lone proton at rest cannot decay into a neutron, a positron (identical to an electron, except for charge) and a neutrino. Explain why your numerical result implies your conclusion.

\[ p \rightarrow n + e^+ + \nu \]

\[ E_f - E_i = m_n c^2 + K_n + m_e c^2 + K_e + K_{\nu} - m_p c^2 \]

\[ = (939.6 \text{ MeV} + K_n + 0.5 \text{ MeV} + K_e + K_{\nu}) - (938.3 \text{ MeV}) \]

\[ = 1.8 \text{ MeV} + (K) \]

If energy is conserved:

\[ 0 = 1.8 \text{ MeV} + K + \nu \]

\[ K = -1.8 \text{ MeV} \]

Since kinetic energy is \( > 0 \), this \textit{rxn} is not expected to occur.
2) **(10 points)** The figure shows the path I take when I make a left turn, taking my daughter to school in the morning. I coast through the turn (no gas, no breaks – clutch depressed, if you know what that is). At the beginning of the turn, my speed is about 13 m/s and at the end (by the arrowhead), my speed is about 7 m/s.

Draw a vector at the halfway point (by the dot) to represent the force on the vehicle. Explain why you drew the vector the way you did.

Vehicle **slows**, so there is a component of force opposite the motion.

Vehicle turns, so there is a component **perpendicular** to the motion.

Intuitively, the perpendicular component is larger than the parallel component. Why?

Quantitatively, $F_{\text{perp}} \approx 20\, \text{kN}$, $F_{\text{para}} \approx 8\, \text{kN}$
3) **(20 points)** Consider an electron traveling at half the speed of light \(v = 0.5c = 1.5 \cdot 10^8 \text{ m/s}\).

A) Use the approximate formula, \(K = \frac{1}{2}mv^2\), to calculate the kinetic energy.

\[
K = \frac{1}{2}mv^2 = \frac{1}{2}(mc^2)^2 = mc^2(0.5)^2 = 0.125 \text{ (mc}^2\text{)} = 0.0625 \text{ MeV} = 10^{-14} \text{ J}
\]

B) Use the relativistically correct formula, \(K = (\gamma - 1)mc^2\), to calculate the kinetic energy.

\[
K = (\gamma - 1)(mc^2) = (\frac{1}{\sqrt{1-0.25}} - 1)(mc^2)
\]

\[
= 0.155 \text{ (mc}^2\text{)} = 0.0774 \text{ MeV} = 1.24 \cdot 10^{-14} \text{ J}
\]
4) **(15 points)** As we discussed in class, we can model the orbit of a planet using a potential of the form: \[ U(r) = U_{\text{circle}} \left( \frac{a^2}{r^2} - 2 \frac{a}{r} \right) \] where \( U_{\text{circle}} \) is the binding energy of a circular orbit, when \( r = a \). Consider an orbit with energy of \( E = -U_{\text{circle}} + \Delta \) (that is, an energy \( \Delta \) greater than the circular orbit). Show that the turning points of the radius of the orbit (that is, the point of closest and farthest approach) are given by:

\[
\frac{r}{a} = 1 \pm \frac{\Delta}{\sqrt{U_{\text{circle}}}}
\]

so

\[-U_{\text{circle}} + \Delta = U_{\text{circ}} \left( \frac{a^2}{r^2} - 2 \frac{a}{r} \right)\]

\[
\left( \frac{a}{r} \right)^2 - 2 \left( \frac{a}{r} \right) + \left( 1 - \frac{a}{U_{\text{circ}}} \right) = 0
\]

\[
\left( \frac{a}{r} \right) = \frac{2 \pm \sqrt{\left( 2^2 - 4 \left( 1 - \frac{a}{U_{\text{circ}}} \right) \right)}}{2(1)}
\]

\[
= 1 \pm \sqrt{\left( 1 - \frac{a}{U_{\text{circ}}} \right)} = 1 \pm \sqrt{\frac{\Delta}{U_{\text{circ}}}}
\]

\[
\frac{r}{a} = \frac{1}{1 \pm \sqrt{\frac{\Delta}{U_{\text{circ}}}}}
\]
5) **(10 points)** When running on level ground, a person must provide energy of about $13.6 \text{ J/m}$ (that is, you burn $13.6$ Joules for each meter you run). But to get a REAL workout, you can go to a stadium and run up the stands.

A) Use PHYSICAL arguments to explain why you burn more calories when you run up stairs of any kind.

Since you increase "height," you increase the Grav. Pot. Energy of the system. If no external agent (like an elevator) does work, it must be done by internal forces (you).

B) Calculate QUANTITATIVELY how much energy you must provide to run up to the top of the stands in Tiger stadium (the top seats are about 54 m above the bottom).

$$\Delta U = mg \Delta y = (75 \text{ kg}) \times (10 \text{ N/kg}) \times (54 \text{ m})$$

$$= 40.5 \text{ kJ} \times \left( \frac{1 \text{ cal}}{4.2 \text{ J}} \right)$$

$$= 9.6 \text{ kcal} \text{ (very disappointing)}$$

(although since humans have an efficiency of about $20\%$, you actually burn more)
6) (30 points) In the 1985 book "Footfall", Larry Niven and Jerry Pournelle write of a group of space-faring elephant-like beings who try to colonize the Earth. One tactic they use is to drop rocks from the asteroid belt onto the Earth. The asteroid belt is about 3 AU from the Sun (where 1 AU is the radius of the Earth's orbit).

A) Assume an average asteroid from the belt (\( m = 10^{15} \text{ kg} \)) starts at rest and falls toward the SUN, calculate the kinetic energy as it crosses the Earth's orbit. (That is, consider only the energy gained by interacting with the Sun).

\[
\Delta K = -\Delta U = - \left[ \frac{GMm}{r_f} + \frac{GMm}{r_i} \right] = GMm \left[ \frac{1}{r_f} - \frac{1}{r_i} \right]
\]

\[
= \left( 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \right) \left( 2 \times 10^{30} \text{ kg} \right) \left( 10^{15} \text{ kg} \right) \left[ \frac{1}{1 \text{ AU}} - \frac{1}{3 \text{ AU}} \right] \frac{1 \text{ AU}}{150 \times 10^8 \text{ m}}
\]

\[
= 6 \times 10^{23} \text{ J}
\]

B) Calculate the ADDITIONAL kinetic energy gained from interacting with the Earth as it approaches the Earth's surface.

\[
\Delta K = -\Delta U = - \left[ \frac{GMm}{r_f} + \frac{GMm}{r_i} \right]
\]

\[
= \left( 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \right) \left( 6 \times 10^{24} \text{ kg} \right) \left( 10^{15} \text{ kg} \right) \left[ \frac{1}{6.4 \times 10^6 \text{ m}} - \frac{1}{450 \times 10^8 \text{ m}} \right]
\]

\[
= (4 \times 10^{29} \text{ J}) \left[ 1.56 \times 10^{-7} - 2.2 \times 10^{-12} \right]
\]

\[
= 6.25 \times 10^{22} \text{ J}
\]