\[ P_{tot} = K_{tot} = K_{trans} + K_{rel} \]

\[ \vec{P}_{tot} = \sum m_i \vec{v}_i = 2\text{kg} \begin{pmatrix} 8 \\ -10 \\ 15 \end{pmatrix} \text{ m/s} + 6\text{kg} \begin{pmatrix} -12 \\ 9 \\ -6 \end{pmatrix} \text{ m/s} + 4\text{kg} \begin{pmatrix} -24 \\ 34 \\ 23 \end{pmatrix} \text{ m/s} \]

\[ = \begin{pmatrix} -152 \\ 158 \\ 86 \end{pmatrix} \text{ Allains} \]

\[ = M_{tot} \vec{V}_{cm} \]

\[ \Rightarrow \vec{V}_{cm} = \begin{pmatrix} -12.67 \\ 13.16 \\ 7.16 \end{pmatrix} \text{ m/s} \]

\[ K_{\text{trans}} = \frac{1}{2} M_{tot} \left| \vec{V} \right|^2 \]

\[ = 6\text{kg} \begin{pmatrix} -12.6 \\ 13.16 \\ 7.16 \end{pmatrix} \cdot \begin{pmatrix} -12.6 \\ 13.16 \\ 7.16 \end{pmatrix} \]

\[ = 2311 \text{ J} \]
Do you get the same result from $K = \frac{P_{tot}}{2M_{tot}}$?

$$K_{tot} = \frac{1}{2} \Sigma M_i v_i^2$$

$$= \frac{1}{2} (2\text{kg})(545\text{ m}^2/\text{s}^2)$$

$$+ \frac{1}{2} (6\text{kg})(261\text{ m}^2/\text{s}^2)$$

$$+ \frac{1}{2} (4\text{kg})(2261\text{ m}^2/\text{s}^2)$$

$$= 5850\text{ J}$$

So $K_q = 5850\text{J} - 2311\text{J}$

$$= 3539\text{ J}$$
Is this the same if we calculate

\[ K_{rel} = \frac{1}{2} m_1 u_1' + \frac{1}{2} m_2 u_2' + \frac{1}{2} m_3 u_3' \]

ANS:

\[ u_1' = u_1 - u_{cm} \]

\[ = (20.6, -29.16, +7.83) \text{ m/s} \]

\[ u_2' = (+0.6, 4.16, -13.16) \text{ m/s} \]

\[ u_3' = (-11.3, +20.83, +15.83) \text{ m/s} \]

\[ K_{rel} = \frac{1}{2} (2 \text{ kg})(1339.16) \text{ m}^2/\text{s}^2 + \frac{1}{2} (6 \text{ kg})(191.16) \text{ m}^2/\text{s}^2 + \frac{1}{2} (4 \text{ kg})(813.16) \text{ m}^2/\text{s}^2 \]

\[ = 3539 \text{ J} \]
For me, the distance from glotes to head in a tuck is about 70 cm. If I approx. the diver as a sphere of mass 70 kg and diameter 70 cm, then:

\[ I = \frac{2}{5}MR^2 = \frac{2}{5}(0.4)(70 \text{ kg})(0.35 \text{ m})^2 \]

\[ = 3.43 \text{ kg} \cdot \text{m}^2 \]

If, instead, I treat it as a cylinder of height 70 cm and diameter 60 cm

\[ I = \frac{1}{4}MR^2 + \frac{1}{2}ML^2 \]

\[ = 4.43 \text{ kg} \cdot \text{m}^2 \]
In the time to fall ~10 m from a high board \( t = \sqrt{\frac{2x}{g}} = 1.4 \text{ s} \) a diver can make maybe 3.5 revolutions. So

\[
\omega = \frac{(3.5)(2\pi)}{1.4 \text{ s}} = 15.7 \text{ rad/s}
\]

so \( K_{\text{rot}} = \frac{1}{2} I \omega^2 \)

\[
= \frac{1}{2} (3.43 \text{ kg·m}^2)(15.7 \text{ rad/s})^2
= 423 \text{ J}
\]
P40

a) A point particle can only move (kinetic energy) or interact (potential energy). It can't store energy inside.

b) \( \Delta y = 0.2 \text{ m} \)

c) \( F_{\text{net},y} = +130 \text{ N} - 68.6 \text{ N} \)
\[ = 61.4 \text{ N} \]

d) \( \Delta y = 0.2 \text{ m} \)

e) \( W = F_{\text{net}} \Delta y = (61.4 \text{ N})(0.2 \text{ m}) \)
\[ = 12.28 \text{ J} \]

f) \( W = \Delta K \rightarrow v = \sqrt{\frac{2W}{m}} = 1.87 \text{ m/s} \)

g) By considering only the work done by the net force,
we only know the change in COM kinetic energy. We're ignorant of other ways to store energy (internal degrees of freedom). All we really have is a conundrum:

\[ W_{\text{Net force}} = 12.28 \text{ J} \]

\[ W_{\text{Gravity}} = -x(68.6 \text{ N})(0.2 \text{ m}) = -13.72 \]

\[ W_{\text{Hand}} = +x(130 \text{ N})(0.8 \text{ m}) = +104.0 \text{ J} \]

\[ 104.0 + (-13.72) \neq +12.28 \]
External degrees of freedom are: \( K_{\text{trans}}, U_{\text{system}} \) on Earth.

Since the author explicitly mentions the string, there is a \( K \) for the string, which is not moving at the same speed as the box.

In the time the box moves 0.2 m, the COM of the string moved 0.2 m + \( \frac{1}{2} \times 0.6 \) m = 0.5 m. So \( U_{\text{string}} = 2.5 U_{\text{box}} \).

If \( m_{\text{string}} \ll m_{\text{box}} \), we might be able to ignore this.

If \( m_{\text{string}} = 0.05 m_{\text{box}} \), show \( K_{\text{string}} = 0.25 K_{\text{box}} \).
Similarly, there is a string and a Earth string \((0.2 + \frac{1}{2}0.6)\) m

\( \text{Show (if } M_{\text{string}} = 0.05 \text{ M}_{\text{box}} \text{)} \)

\[
\frac{U_{\text{string}}}{U_{\text{Earth}}} = 0.125 \frac{U_{\text{box}}}{U_{\text{Earth}}}
\]

i) From part e), \( K = 12.28 \text{ J} \)

j) as d), if we ignore the string as small

[Notes: In parts a) \(\rightarrow\) g) we also assumed \(M_{\text{string}} \) small enough to ignore. If we don't, those answers would be different.]
(40 k) \[ W_{\text{grav}} = (mg)(Ay)\cos\theta \]
\[ = (68.6 N)(0.2 m)(-1) \]
\[ = -13.72 \text{ J} \]

(2) \[ \Delta x_{\text{total}} = 0.8 \text{ m} \]

(m) \[ W = F\Delta x \cdot \cos\theta = (130 N)(0.8 m)(1) \]
\[ = 104 \text{ J} \]

(n) I would say rather, by how much has the energy of the system changed:
\[ +104 \text{ J} + (-13.72 \text{ J}) = 90.28 \text{ J} \]

In this case, it's all K, some external (K_extn), some internal (K_int). Generally, it could be if
We've seen examples in nuclear reactions where energy showed up as inertial mass. The overriding principle is; if some energy seems to be missing:

\[
\text{Work} = +104 \text{J} + (-13.72 \text{J}) = 90.28 \text{J} > K_{\text{trans}}.
\]

Then go look for it:

1) \[ K_{\text{rel}} = 90.28 \text{J} - 12.28 \text{J} = 78 \text{J} \]
Note well that your author and I have often used "d" in this context as a distance = bond length. So I'll use D for the distance the box moves macroscopically (the distance I can measure with my ruler).

e) $F_{net, x} = \mu N = \mu mg$

$$a_x = \frac{F_{net, x}}{m} = \mu g$$

$$v^2 = 2a_xD = 2\mu g D$$

$$D = \frac{v^2}{2\mu g}$$

$$\frac{v^2}{2} = W_{Net, x} = (\mu mg)D = \frac{1}{2}mv^2$$

$$D = \frac{v^2}{2\mu g}$$
Wow I have to use $F = ma$ because energy won't give time: $D = \frac{1}{2}at^2$

\[ t = \sqrt{\frac{2D}{a}} = \sqrt{\frac{2u^2}{(2\mu g)^2}} \]

\[ = \frac{u}{\mu g} \text{ (as expected).} \]

c) Generally, if the system heats up, the friction force has to do more work than appears as $K$-loss. So it must act through a larger effective distance. Let's
consider our favorite model of atomic interactions—tiny little springs—

the spring stretches a distance larger than the box moves. When the spring breaks, the extra work slows up as heat.