1) (10 points) Data from the newly-commissioned Transiting Exoplanet Survey Satellite (TESS) show a planet around the star LHS 3844, named LHS 3844 b. It has a radius 1.32 times bigger than Earth's (so \( R = 8.45 \times 10^8 \text{ m} \)).

A) Assuming (for no particular reason) that the density is the same as Earth's: \( \rho = 5500 \text{ kg/m}^3 \) calculate the mass of LHS 3844 b.

\[
M = \rho V = \left( 5500 \frac{\text{kg}}{\text{m}^3} \right) \left( \frac{4}{3} \pi \right) \left( 8.45 \times 10^8 \text{ m} \right)^3
\]

\[
= 1.3 \times 10^{25} \text{ kg} \quad (\approx 2 \times M_{\text{Earth}})
\]

B) Calculate the strength of the gravitational field at the surface of LHS 3844 b.

\[
g = \frac{GM}{R^2} = \frac{\left( 4.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \right) \left( 1.3 \times 10^{25} \text{ kg} \right) \left( 8.45 \times 10^8 \text{ m} \right)}{(8.45 \times 10^8 \text{ m})^2} = 12 \frac{\text{N}}{\text{kg}}
\]

\[
a \cdot g = \frac{GM}{R^2} = \frac{G \cdot \frac{4}{3} \pi R^3 \cdot \rho}{\alpha^2} = \frac{4}{3} \pi G \rho R
\]

\[
= 12 \frac{\text{N}}{\text{kg}}
\]
2) (10 points) At the end of Chap. 2, we discussed the 1D motion of a particle subject to a constant force, $F$. Since $F = \frac{dp}{dt}$, this means that $p$ will increase linearly with time. The graph shows a plot of the speed of such an object as a function of time. Recall that generally,

$$p = \gamma mv.$$  

A) Consider the initial part of the graph (where the speed rises). Explain why this is what is expected for a constant force (HINT: what is momentum when $v \ll c$?).

$$\frac{dp}{dt} = m \left( \frac{dv}{dt} + v \frac{d\gamma}{dt} \right)$$

@ low speeds, $\gamma \approx 1 = \text{const}$

$\therefore \frac{d\gamma}{dt} = 0$

$\therefore \frac{dp}{dt} \approx m \frac{dv}{dt}$. Then $v \text{ will increase with } t$, or $v = at$

B) Consider the later part of the graph (where the speed no longer rises). Explain why this is what is expected for a constant force.

In class, we showed that for long times, $v \approx c$

That is, $v \approx \text{constant}$, so $\frac{dv}{dt} \approx 0$. Then $\frac{dp}{dt} \approx mc \frac{d\gamma}{dt}$

$\therefore$ In fact, it should be possible to show that

$$\gamma = \frac{at}{c}$$

C) On the graph, sketch what the speed would look like if the force were half as big. Explain why you drew the graph the way you did.

the system will still approach the same speed, $(v \approx c)$. But the initial slope ($\frac{v}{m}$) will be $\frac{1}{2}$ as big.
3) **(30 points)** A hydrogen molecule can be ionized (one of the two electrons removed). Although it seldom happens on Earth, it's important for the chemistry of interstellar clouds.

A) Sketch the “hydrogen molecular ion” by putting a proton at the origin, \((0,0)\), its electron at \((50,0)\) pm, and another proton at \((100,0)\) pm.

![Diagram of hydrogen molecular ion]

B) Calculate the force the hydrogen atom (on the x axis) exerts upon the ion (the lone proton).

\[
\frac{1}{a_3} = \frac{kq_1 q_3}{r_{13}^2} \langle 1,0 \rangle + \frac{kq_2 q_3}{r_{23}^2} \langle 1,0 \rangle
\]

\[
= \frac{(9.10^{-8}N \cdot m^2)}{e^2} \left( 1.6 \times 10^{-19} \right)^2 \left( \frac{(2\times1)(1\times1)}{4} \right)
\]

\[
= (9.10^{-8}N) \langle -\frac{3}{4},0 \rangle
\]

\[
= 6.9 \times 10^{-8}N \langle -1,0 \rangle
\]
4) **(20 points)** When you stand on a concrete sidewalk, the concrete compresses enough for the concrete to "know" how hard to push up on you. The Young's modulus for concrete varies according to composition, but a typical value is about 40 GPa. \( \frac{F}{A} = \frac{Y}{L} \)

A) Calculate the amount (\( \Delta L \)) the concrete compresses when you stand on it. Tell me any numbers you estimate.

My shoe is roughly rectangular: \( A = (0.23 \text{ m})(0.1 \text{ m}) = 0.023 \text{ m}^2 \)

My weight \( W = mg = (75 \text{ kg})(10 \text{ m/s}^2) = 750 \text{ N} \)

\[
\frac{750 \text{ N}}{2(0.03 \text{ m}^2)} = (40 \text{ GPa})(\frac{\Delta L}{0.3 \text{ m}}) -
\]

I estimate that a sidewalk is about 1 ft = 0.3 m thick.

\[
\Delta L = (0.3 \text{ m})(12,500 \text{ Pa}) = 9 \times 10^{-8} \text{ m} = 9 \times 10^{-7} \text{ m}
\]

B) Knowing that concrete FAILS when you apply a stress of about 30 MPa, calculate the hardest force you can exert on concrete before it fails.

\[
\frac{F}{A} = \frac{F}{2(0.03 \text{ m}^2)} \leq 30 \times 10^6 \text{ Pa}
\]

\[
F \leq 1.8 \times 10^6 \text{ N}
\]
5) (15 points) Imagine that a uniform stick of length, $L$, and mass, $M$, is bent so that $1/4$ of the length is along the x axis and $3/4$ is along the y axis (with the corner at the origin). Find the location of the center of mass.

$$M \hat{r}_{cm} = \frac{M}{4} \begin{pmatrix} \frac{L}{8} \\ 0 \end{pmatrix} + \frac{3}{4} M \begin{pmatrix} 0 \\ \frac{3L}{8} \end{pmatrix}$$

$$= ML \begin{pmatrix} \frac{L}{82} \\ 0 \end{pmatrix} + ML \begin{pmatrix} 0 \\ \frac{9}{82} \end{pmatrix}$$

$$= ML \begin{pmatrix} \frac{L}{82} \\ \frac{9}{82} \end{pmatrix}$$

$$\hat{r}_{cm} = \frac{L}{4} \begin{pmatrix} \frac{1}{8} \\ \frac{9}{8} \end{pmatrix}$$
6) (15 points) We mentioned the carbon nanotube in class, and said that the stiffness of the C-C bond is \( 400 \text{ N/m} \). The bond length is \( 140 \text{ pm} \). Recall that we described a carbon nanotube as a wire with 12 chains of atoms (i.e., \( N_{\text{chains}} = 12 \)).

A) Calculate Young's modulus for a carbon nanotube.

\[
Y = \left( \frac{k_{\text{s}}}{d} \right) = \frac{400 \text{ N/m}}{140 \times 10^{-12} \text{ m}} = 2.860 \text{ GPa}
\]

B) If the bond length can stretch by 10% before breaking, how much weight can a 0.5 m long nanotube support.

\[
A = 12d^2 = 12 \left( 140 \times 10^{-12} \text{ m} \right)^2 = 2.35 \times 10^{-19} \text{ m}^2
\]

\[
\frac{F}{A} = Y \frac{AL}{L}
\]

\[
\rightarrow F = AY \frac{AL}{L} = \left( 2.35 \times 10^{-19} \text{ m}^2 \right) \left( 2.860 \times 10^9 \frac{\text{N}}{\text{m}^2} \right) \left( 0.1 \right)
\]

\[
= 6.7 \times 10^{-8} \text{ N}
\]