1) A wire of length L and uniform linear density is bent to form three sides of a square.
   A) Find the location of the center of mass of this bent wire.

   \[ M_{y_{cm}} = \frac{M}{3} \left(\frac{L}{6}\right) + \frac{M}{3} \left(\frac{L}{3}\right) + \frac{M}{3} \left(\frac{L}{6}\right) = \frac{4}{18} ML = \frac{2}{9} ML \]

   \[ y_{cm} = \frac{2}{9} L \]

   \[ M_{x_{cm}} = \frac{M}{3} \left(-\frac{L}{6}\right) + \frac{M}{3} (0) + \frac{M}{3} \left(\frac{L}{6}\right) = 0 \]

   \[ x_{cm} = 0 \]

   B) The wire is then reformed into a square. Find the moment of inertia about an axis perpendicular to the square and through the center of mass.

   \[ I = \frac{1}{12} (\frac{M}{4}) \left(\frac{L}{4}\right)^2 + M \left(\frac{L}{8}\right)^2 \]

   \[ = \frac{1}{12} \cdot \frac{ML^2}{64} + \frac{ML^2}{64} = \frac{13}{12} \cdot \frac{ML^2}{64} \]

   \[ I_{tot} = 4 \cdot I_{each} = \frac{13}{12} \cdot \frac{ML^2}{16} \]
2) We discussed in class two ways to catch an egg without breaking it: a) begin with my arm stretched out in front and draw my arm straight back until it is stretched out behind me or b) begin with my arm stretched out in front and rotate my arm in a half circle until it is stretched out behind me. Consider that in the former case there is only a tangential force over a shorter distance and that in the latter, there is a longer distance, but there is a centripetal force as well as a tangential force.

Argue which approach is better if the goal is to avoid breaking the egg. If it's impossible to say, explain why.

\[ F_{t} = \frac{m v_0}{\pi t} = \frac{m v_0}{2D(v_0)} = \frac{m v_0^2}{2D} \]

The centripetal force will be:

\[ F = \frac{m v_0^2}{r} = \frac{m}{L} \left( \frac{v_0}{2} \right)^2 = \frac{m v_0^2}{4L} \]

where \( L \) is the length of the arm.

**Linear case**: \( F_c = 0 \), so \( F_{net} = F_t \)

\[ F_{net} = \frac{m v_0^2}{2(2L)} = \frac{m v_0^2}{4L} \]

**\( \frac{1}{2} \) Circle case**: \( F_{net} = \left[ F_{t}^2 + F_c^2 \right]^{1/2} \)

\[ \left( \frac{m v_0^2}{4L} \right) \left[ \frac{4}{\pi^2} + 1 \right]^{1/2} \]

So Linear is better.
3) In one of your homework problems, you calculated at what angle a rolling object will fly off when rolling from the top of a circular object. You calculated that the rolling object will fly off at the angle given by:

\[ \cos(\theta_{\text{MAX}}) = \frac{2}{3 + \left( \frac{I}{mr^2} \right)} \]

where \( I \) is the moment of inertia of the rolling object and \( m \) and \( r \) are its mass and radius, respectively. The problem referred to a 1 cm radius marble rolling on a large sphere. In that problem, the angle \( \theta_{\text{MAX}} \) calculates to 54°.

A) What does the angle become if a steel ball bearing of radius 3 cm is substituted?

-Same: \( \frac{I}{mr^2} \) depends only on \( m \) and \( r \) on geometry, not on size.

B) What is the angle if a ring is rolled off the sphere?

\[ \cos(\theta) = \frac{2}{3 + 1} = \frac{1}{2}, \text{ so} \]

\[ \theta = 60^\circ \]
4) We calculate that, using the vector cross product, we can write the centripetal acceleration as: \( \vec{a}_c = \vec{\omega} \times \vec{\omega} \times \vec{r} \) and: \( \vec{v} = \vec{\omega} \times \vec{r} \). We can combine these and get: 
\[
\vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r})
\]

A) Are the parentheses necessary? That is, if you don't have the parentheses there, is the order of calculation ambiguous?

\( \text{NO: if I write } \vec{a}_c = \vec{\omega} \times \vec{\omega} \times \vec{r} \text{ and calculate } \vec{a}_c = (\vec{\omega} \times \vec{\omega}) \times \vec{r}, \text{ then } \vec{a}_c = \vec{0} \text{ always which is ridiculous.} \)

B) Calculate the magnitude and direction of this acceleration for an object on the Earth's equator.

\[
|\vec{a}_c| = \omega^2 r = \left(\frac{4\pi^2}{T^2}\right) r
\]

\[
= \frac{4\pi^2}{(86,400)^2} \times (6.4 \times 10^6 \text{m/s}) = 0.034 \text{ m/s}^2
\]

C) Calculate the magnitude and direction of this acceleration for an object on the Earth's pole.

\[
\vec{\omega} \times \vec{r} = 0, \text{ so } \vec{a}_c = \vec{0}
\]
5) If the Earth were compressed small enough for the escape velocity to be that of the speed of light, it would then be a black hole. There are a few things that are preserved when an object becomes a black hole and one of them is angular momentum. As we will see in Chap 11, if the Earth were a black hole, its radius would be about 1 cm. If the angular momentum is preserved, what would the period of rotation be? (You may assume that the black hole is a uniform density sphere).

\[ L = I \omega = \text{const.} = \frac{2\pi I}{T} \]

\[ \frac{2\pi I}{T} = \frac{2\pi I}{T} \Rightarrow T_{BH} = T_E \left( \frac{I_{BH}}{I_E} \right) \]

\[ T_{BH} = T_E \left( \frac{\frac{\rho M R^2}{\frac{2}{5} MR^2}}{\frac{\rho M R^2}{\frac{2}{5} MR^2}} \right) = (86,400) \left( \frac{10^2 m}{6.4 \times 10^6 m} \right)^2 \]

\[ = 2 \times 10^{-13} \text{ sec} \]