In class, we were unable to reconcile two calculations of the cyclotron radius of a proton heading at us from the sun. (The cyclotron radius is the fancy name for the radius of the path of a charged particle takes when travelling through a magnetic field).

Recall that we supposed that a proton heading at us from the sun at a speed about equal to the speed of light (this is fast, but not particularly rare). I assumed it was noon in Hammond on one of the equinoxes. The sun is then directly over the equator and about $60^\circ$ above the horizon as viewed in Hammond. So a proton travelling directly at us (from the Sun) will have a velocity $60^\circ$ BELOW the horizon. I’ve measured the magnetic field direction in Pursley Hall, and it’s about $45^\circ$ below the horizontal. So the velocity will make an angle of $15^\circ$ with the magnetic field. I can use all this to find the magnitude of the magnetic force on this proton:

$$|\vec{F}_B| = qvB \sin \theta = (1.6 \cdot 10^{-19} \text{ C}) (3 \cdot 10^8 \text{ m/s}) (50 \cdot 10^{-6} \text{ T}) \sin (15^\circ) = 6.2 \cdot 10^{-16} \text{ N}$$

Recall that I then calculated the acceleration produced by this force:

$$a = \frac{F}{m} = \frac{6.2 \cdot 10^{-16} \text{ N}}{1.67 \cdot 10^{-27} \text{ kg}} = 3.7 \cdot 10^{11} \text{ m/s}^2$$

So far so good (but this is where everything went off the rails).

Recall that if a particle’s velocity is parallel to the field, then there is no force and no acceleration. And if the velocity is perpendicular to the field, then the force is maximal. But our situation is in between. And it’s easy enough to deal with this – just consider the two components of the velocity. The component parallel to the field is: $v_{\text{para}} = (3 \cdot 10^8 \text{ m/s}) \cos 15^\circ = 2.90 \cdot 10^8 \text{ m/s}$ But there is no component of the magnetic force in this direction, so this component of the velocity will NOT change. The component perpendicular to the magnetic field is: $v_{\text{perp}} = (3 \cdot 10^8 \text{ m/s}) \sin 15^\circ = 0.78 \cdot 10^8 \text{ m/s}$ and this is the component that will change. It will change direction to make the circle we discussed in class. So if I use this in the calculation we did first:

$$r = \frac{v_{\text{perp}}^2}{a_{\text{cent}}} = \frac{(0.78 \cdot 10^8 \text{ m/s})^2}{3.7 \cdot 10^{11} \text{ m/s}^2/s} = 16.3 \text{ km}$$

and in the calculation we did second:

$$r = \frac{mv_{\text{perp}}}{qB} = \frac{(1.67 \cdot 10^{-27} \text{ kg})(0.78 \cdot 10^8 \text{ m/s})}{(1.6 \cdot 10^{-19} \text{ C})(50 \cdot 10^{-6} \text{ T})} = 16.3 \text{ km}$$

you get the same result (whew).

As stressful as this has been, note that the correct answer (above) is different from BOTH numbers we got in class. Had we not tried both ways, we would not have caught the mistake. Keep this in mind when you’re trying to solve a problem that you don’t know the answer to (that maybe NO ONE knows the answer to) – this technique (using multiple methods that should give the same result) is one of the ways to check.