Chapter 20: Electromagnetic Induction

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20.1 Motional Emf Voltage

In this section we study **motional emf voltage** — the emf voltage induced when a conductor is moved in a magnetic field.

Motional emf voltage is the principle behind the electric generator.

Imagine a metal rod of length \( L \) in a uniform magnetic field.
Motional Emf Voltage 2

When the rod is at rest, the conduction electrons move in random directions at high speeds, but their average velocity is zero.

Since their average velocity is zero, the average magnetic force on the electrons is zero; therefore, the total magnetic force on the rod is zero.

The magnetic field affects the motion of individual electrons, but the rod as a whole feels no net magnetic force.
Now consider a rod that is moving instead of being at rest.

The average magnetic force on each conduction electron is

\[ \mathbf{F}_B = -e \mathbf{v} \times \mathbf{B} \]
The magnetic force causes electrons to accumulate at the lower end, giving it a negative charge and leaving positive charge at the upper end.

As charge accumulates at the ends, an electric field develops in the rod, with field lines running from the positive to the negative charge.
Motional Emf Voltage 5

Eventually an equilibrium is reached: the electric field builds up until it causes a force equal and opposite to the magnetic force on electrons in the middle of the rod.

Then there is no further accumulation of charge at the ends. Thus, in equilibrium,

\[ \vec{F}_E = q \vec{E} = -\vec{F}_B = -\left( q \vec{v} \times \vec{B} \right) \]

\[ \vec{E} = -\vec{v} \times \vec{B} \]

Since \( \vec{v} \) and \( \vec{B} \) are perpendicular, \( E = vB \).

\[ \Delta V = EL = vBL \]
Motional Emf Voltage 6

In this case, the direction of $\mathbf{E}$ is parallel to the rod. If it were not, then the potential difference between the ends is found using only the component of $\mathbf{E}$ parallel to the rod:

$$\Delta V = E_\parallel L$$

As long as the rod keeps moving at constant speed, the separation of charge is maintained. The moving rod acts like a battery that is not connected to a circuit; positive charge accumulates at one terminal and negative charge at the other, maintaining a constant potential difference.
The resistor R sees a potential difference $\Delta V$ across it, so current flows.

The current tends to deplete the accumulated charge at the ends of the rod, but the magnetic force pumps more charge to maintain a constant potential difference.
Motional $\textbf{Emf}$ Voltage 8

Motional $\textbf{emf}$ voltage:

$$\mathcal{E} = vBL$$

More generally, if $\vec{E}$ is not parallel to the rod, then

$$\mathcal{E} = (\vec{v} \times \vec{B}) \parallel L$$
Motional Emf Voltage 9

Where does the electric energy come from? The rod is acting like a battery, supplying electric energy that is dissipated in the resistor. How can energy be conserved?

As soon as current flows through the rod, a magnetic force acts on the rod in the direction opposite to the velocity.

Left on its own, the rod would slow down as its kinetic energy gets transformed into electric energy.
To maintain a constant emf voltage, the rod must maintain a constant velocity, which can only happen if some other force pulls the rod.

The work done by the force pulling the rod is the source of the electric energy.
Example 20.1

A square metal loop made of four rods of length $L$ moves at constant velocity $v$. The magnetic field in the central region has magnitude $B$; elsewhere the magnetic field is zero. The loop has resistance $R$.

At each position 1–5, state the direction (CW or CCW) and the magnitude of the current in the loop.
Example 20.1 Strategy

If current flows in the loop, it is due to the motional $\text{emf}$ voltage that pumps charge around. The vertical sides ($a$, $c$) have motional $\text{emf}$ voltages as they move through the magnetic field.

We need to look at the horizontal sides ($b$, $d$) to see whether they also give rise to motional $\text{emf}$ voltages. Once we figure out the $\text{emf}$ voltage in each side, then we can determine whether they cooperate with each other—pumping charge around in the same direction—or tend to cancel each other.
Example 20.1 Solution 1

The vertical sides \((a, c)\) have motional \(\text{emf}\) voltages as they move through the region of magnetic field. The \(\text{emf}\) voltage acts to pump current upward (toward the top end).

The magnitude of the \(\text{emf}\) voltage is

\[
\mathcal{E} = vBL
\]
Example 20.1 Solution 2

For the horizontal sides \((b, d)\), the average magnetic force on a current-carrying electron is

\[
\vec{F}_{av} = -e \vec{v} \times \vec{B}
\]

Since the velocity is to the right and the field is into the page, the right-hand rule shows that the direction of the force is down, just as in sides \(a\) and \(c\).

However, now the magnetic force does not move charge along the length of the rod; the magnetic force instead moves charge across the diameter of the rod.
Example 20.1 Solution 3

An electric field then develops across the rod. In equilibrium, the magnetic and electric forces cancel, exactly as in the Hall effect.

The magnetic force does not push charge along the length of the rod, so there is no motional emf voltage in sides $b$ and $d$.

In positions 1 and 5, the loop is completely out of the region of magnetic field. There is no motional emf voltage in any of the sides; no current flows.
Example 20.1 Solution 4

In position 2, there is a motional $\varepsilon$ voltage in side $c$ only; side $a$ is still outside the region of $\mathbf{B}$. The $\varepsilon$ voltage makes current flow upward in side $c$, and therefore counterclockwise in the loop.

The magnitude of the current is

$$I = \frac{\varepsilon}{R} = \frac{\nu BL}{R}$$
Example 20.1 Solution 5

In position 3, there are motional emf voltages in both sides $a$ and $c$. Since the emf voltages in both sides push current toward the top of the loop, the net emf voltage around the loop is zero—as if two identical batteries were connected. No current flows around the loop.
Example 20.1 Solution 6

In position 4, there is a motional \textbf{emf} voltage only in side \textit{a}, since side \textit{c} has left the region of the \textbf{B} field. The \textbf{emf} voltage makes current flow upward in side \textit{a}, and therefore \textit{clockwise} in the loop. The magnitude of the current is again

\[ I = \frac{\mathcal{E}}{R} = \frac{vBL}{R} \]
20.2 Electric Generators
None of the four sides of the loop moves perpendicularly to the magnetic field at all times, so we must generalize the results of Section 20.1.

In Problem 72, you can verify that there is zero induced emf voltage in sides 1 and 3, so we concentrate on sides 2 and 4.

Since these two sides do not, in general, move perpendicularly to \( \mathbf{B} \), the magnitude of the average magnetic force on the electrons is reduced by a factor of \( \sin \theta \), where \( \theta \) is the angle between the velocity of the wire and the magnetic field:

\[
F_{av} = evB \sin \theta
\]
Electric Generators 3

\[ \% = \nu BL \sin \theta \]

The induced \textit{emf} voltage is then reduced by the same factor:

\[ \mathcal{E} = (\vec{v} \times \vec{B})_\parallel L = \nu Bl \sin \theta = \nu_\perp BL \]

Note that the induced \textit{emf} voltage is proportional to the component of the velocity perpendicular to \( B \) ( \( \nu_\perp = \nu \sin \theta \) ).

For a visual image, think of the induced \textit{emf} voltage as proportional to the rate at which the wire \textit{cuts through magnetic field lines}. The component of the velocity \textit{parallel} to \( B \) moves the wire along the magnetic field lines, so it does not contribute to the rate at which the wire cuts through the field lines.
Electric Generators 4

The loop turns at constant angular speed $\omega$, so the speed of sides 2 and 4 is

$$v = \omega r$$

The angle $\theta$ changes at a constant rate $\omega$. For simplicity, we choose $\theta = 0$ at $t = 0$, so that $\theta = \omega t$ and the emf voltage $\mathcal{E}$ as a function of time $t$ in each of sides 2 and 4 is

$$\mathcal{E}(t) = vBl \sin \theta = \omega r Bl \sin(\omega t)$$
Electric Generators 5

Sides 2 and 4 move in opposite directions, so current flows in opposite directions; in side 2, current flows into the page, while in side 4 it flows out of the page.
Electric Generators 6

*Both* sides tend to send current counterclockwise around the loop as viewed in the figure. Therefore, the *total* emf voltage in the loop is the *sum* of the two:

\[ \mathcal{E} (t) = 2 \omega r B L \sin(\omega t) \]

The rectangular loop has sides \( L \) and \( 2r \), so the area of the loop is \( A = 2rL \). Therefore, the total emf voltage \( \mathcal{E} \) as a function of time \( t \) is

\[ \mathcal{E} (t) = \omega BA \sin(\omega t) \]
Electric Generators 7

When written in terms of the area of the loop,

\[ \mathcal{E}(t) = \omega BA \sin(\omega t) \]

is true for a planar loop of any shape.

If the coil consists of \( N \) turns of wire (\( N \) identical loops), the emf voltage is \( N \) times as great.

**Emf Voltage produced by an ac generator:**

\[ \mathcal{E}(t) = \omega N BA \sin(\omega t) \]
The \textbf{emf} voltage produced by a generator is not constant; it is a sinusoidal function of time.

The maximum \textbf{emf} voltage \(( = \omega NBA \) ) is called the \textbf{amplitude} of the \textbf{emf} voltage (just as in simple harmonic motion, where the maximum displacement is called the amplitude).
Application: The DC Generator

A dc generator is one in which the emf voltage does not reverse direction.

One way to make a dc generator is to equip the ac generator with a split-ring commutator and brushes, exactly as for the dc motor (Section 19.7).

Just as the emf voltage is about to change direction, the connections to the rotating loop are switched as the brushes pass over the gap in the split ring. The commutator effectively reverses the connections to the outside load so that the emf voltage and current supplied maintain the same direction.
Application: The DC Generator

The emf voltage and current are not constant, though. The emf voltage is described by:

\[ \mathcal{E}(t) = \omega N BA |\sin(\omega t)| \]

(this is called full-wave rectification – there’s an easier way to do it with diodes – this is how your car changes the AC voltage from the alternator into a DC voltage to charge your phone)
Application: The DC Generator

More sophisticated dc generators have many coils distributed evenly around the axis of rotation. The emf voltage in each coil still varies sinusoidally, but each coil reaches its peak emf voltage at a different time.

As the commutator rotates, the brushes connect selectively to the coil that is nearest its peak emf voltage.

The output emf voltage has only small fluctuations, which can be smoothed out by a circuit called a voltage regulator if necessary.
Example 20.2

A simple dc generator in contact with a bicycle’s tire can be used to generate power for the headlight. The generator has 150 turns of wire in a circular coil of radius 1.8 cm. The magnetic field strength in the region of the coil is 0.20 T. When the generator supplies an emf voltage of amplitude 4.2 V to the lightbulb, the lightbulb consumes an average power of 6.0 W and a maximum instantaneous power of 12.0 W.
Example 20.2 (2)

(a) What is the rotational speed in rpm of the armature of the generator?

(b) What is the average torque and maximum instantaneous torque that must be applied by the bicycle tire to the generator, assuming the generator to be ideal?

The radius of the tire is 32 cm, and the radius of the shaft of the generator where it contacts the tire is 1.0 cm. At what linear speed must the bicycle move to supply an emf voltage of amplitude 4.2 V?
Example 20.2 Solution 1

(a)

\[ \mathcal{E}(t) = \omega \cdot N \cdot BA \sin(\omega t) \]

\[ \mathcal{E}_{\text{MAX}} = \omega \cdot N \cdot BA \]

N = 150, A = \pi r^2, and B = 0.20 \, \text{T}

\[ \omega = \frac{\mathcal{E}_{\text{MAX}}}{NBA} = \frac{4.2 \, \text{V}}{150 \cdot \pi \cdot (0.018 \, \text{m})^2 \cdot 0.20 \, \text{T}} = 137.5 \, \text{rad/s} \]

\[ = \frac{137.5 \, \text{rad}}{\text{s}} \cdot \frac{1 \, \text{rev}}{2 \, \pi \, \text{rad}} \cdot \frac{60 \, \text{s}}{1 \, \text{min}} = 1300 \, \text{rpm} \]
Example 20.2 Solution 2

(b) 

\[ P = \frac{W}{\Delta t} \]

\[ P = \tau \frac{\Delta \theta}{\Delta t} = \tau \omega \]

\[ \tau_{\text{av}} = \frac{P_{\text{av}}}{\omega} = \frac{6.0W}{137.5 \text{rad} / \text{s}} = 0.044 \text{ N} \cdot \text{m} \]

\[ \tau_{\text{m}} = \frac{P_{\text{m}}}{\omega} = \frac{12.0W}{137.5 \text{rad} / \text{s}} = 0.087 \text{ N} \cdot \text{m} \]
Example 20.2 Solution 3

(c) \( v_{\text{tan}} = \omega r = 137.5 \, \text{rad/s} \times 0.010 \, \text{m} = 1.4 \, \text{m/s} \)

Assuming that the bicycle rolls without slipping on the road, its linear speed is approximately 1.4 m/s (about 3 mph – walking speed – so a leisurely ride).
20.3 Faraday’s “Law”.

A Changing Magnetic Field Can Cause an Induced Emf Voltage.

In 1831, Faraday discovered two ways to produce an induced emf voltage.

One is to move a conductor in a magnetic field (motional emf voltage).

The other does not involve movement of the conductor. Instead, Faraday found that a changing magnetic field induces an emf voltage in a conductor even if the conductor is stationary.

The induced emf voltage due to a changing B field cannot be understood in terms of the magnetic force on the conduction electrons: if the conductor is stationary, the average velocity of the electrons is zero, and the average magnetic force is zero.
Consider a circular loop of wire between the poles of an electromagnet. The loop is perpendicular to the magnetic field; field lines cross the interior of the loop.
20.3 Faraday’s “Law”

Since the strength of the magnetic field is related to the spacing of the field lines, if the strength of the field varies (by changing the current in the electromagnet), the number of field lines passing through the conducting loop changes.

Faraday found that the \textit{emf} voltage induced in the loop is proportional to the \textit{rate of change} of the number of field lines that cut through the interior of the loop.
20.3 Faraday’s “Law”.

We can formulate Faraday’s “law” mathematically so that numbers of field lines are not involved.

The magnitude of the magnetic field is proportional to the number of field lines per unit cross-sectional area:

\[ B \propto \frac{\text{number of lines}}{\text{area}} \]

If a flat, open surface of area \( A \) is perpendicular to a uniform magnetic field of magnitude \( B \), then the number of field lines that cross the surface is proportional to \( BA \), since

\[ \text{number of lines} = \frac{\text{number of lines}}{\text{area}} \times \text{area} \propto BA \]
20.3 Faraday’s “Law”

In general, the number of field lines crossing a surface is proportional to the perpendicular component of the field times the area:

\[
\text{number of lines } \propto B_\perp A = BA \cos \theta
\]

If a flat, open surface of area \( A \) is perpendicular to a uniform magnetic field of magnitude \( B \), then the number of field lines that cross the surface is proportional to \( BA \), since
20.3 Faraday’s “Law”

number of lines $\propto B \perp A = BA \cos \theta$
Magnetic Flux

The mathematical quantity that is proportional to the number of field lines cutting through a surface is called the **magnetic flux**.

The symbol $\Phi$ (Greek capital phi) is used for flux; in $\Phi_B$ the subscript $B$ indicates magnetic flux.

**Magnetic flux through a flat surface of area $A$:**

$$\Phi_B = B \cdot A = BA \cdot = BA \cos \theta$$

($\theta$ is the angle between $B$ and the normal to the surface)

---

The SI unit of magnetic flux is the weber ($1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$).
**Statement of Faraday’s “Law”**

Faraday’s “law” says that the magnitude of the induced emf voltage around a loop is equal to the rate of change of the magnetic flux through the loop.

\[ \mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t} \]

If, instead of a single loop of wire, we have a coil of \( N \) turns, then the total emf voltage in the coil is then \( N \) times as great:

\[ \mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t} \]

The quantity \( N \Phi_B \) is called the total flux linkage through the coil.
Example 20.3

A 40.0-turn coil of wire of radius 3.0 cm is placed between the poles of an electromagnet. The field increases from 0 to 0.75 T at a constant rate in a time interval of 225 s.

What is the magnitude of the induced emf voltage in the coil if
(a) the field is perpendicular to the plane of the coil?
(b) the field makes an angle of 30.0° with the plane of the coil?
Example 20.3 Strategy

First we write an expression for the flux through the coil in terms of the field.

The only thing changing is the strength of the field, so the rate of flux change is proportional to the rate of change of the field.

Faraday’s “law” gives the induced emf voltage.
Example 20.3 Solution 1

(a) The flux through one turn is

\[ \Phi_B = BA \]

\[ \mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t} = -N \frac{B_f A - 0}{\Delta t} \]

\[ |\mathcal{E}| = 40.0 \times \frac{0.75 \, T \times \pi \times (0.030 \, m)^2}{225 \, s} = 3.77 \times 10^{-4} \, V \]

\[ = 0.38 \, mV \]
Example 20.3 Solution 2

(b)

\[ \theta = 90.0^\circ - 30.0^\circ = 60.0^\circ \]

\[ \Phi_B = BA \cos \theta \]

\[ |\mathcal{E}| = N \frac{\Delta \Phi_B}{\Delta t} = N \frac{B_A A \cos \theta - 0}{\Delta t} \]

\[ = 3.77 \times 10^{-4} \text{ V} \times \cos 60.0^\circ \]

\[ = 0.19 \text{ mV} \]
Faraday’s “Law” and Motional $\text{Emf}$ Voltages

Earlier in this section, we wrote Faraday’s “law” to give the magnitude of the induced $\text{emf}$ voltage due to a changing magnetic field.

But that’s only part of the story. Faraday’s “law” gives the induced $\text{emf}$ voltage due to a changing magnetic flux, *no matter what the reason for the flux change.*

The flux change can occur for reasons other than a changing magnetic field. A conducting loop might be moving through regions where the field is not constant, or it can be rotating, or changing size or shape.

In all of these cases, Faraday’s “law” as already stated gives the correct $\text{emf}$ voltage, regardless of why the flux is changing.
**Faraday’s “Law” and Motional Emf Voltages**

Recall that flux can be written

$$\Phi_B = BA \cos \theta$$

Then the flux changes if the magnetic field strength \( B \) changes, or if the area of the loop \( A \) changes, or if the angle between the field and the normal changes.

Faraday’s “law” says that, no matter what the reason for the change in flux, the induced emf voltage is

$$\mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t}$$
Emf Voltages that are sinusoidal (sine or cosine) functions of time, such as in Example 20.2, are common in ac generators, motors, and circuits.

A sinusoidal emf voltage is generated whenever the flux is a sinusoidal function of time. It can be shown that:

If $\Phi(t) = \Phi_0 \sin \omega t$, then $\frac{\Delta \Phi}{\Delta t} = \omega \Phi_0 \cos \omega t$ (for small $\Delta t$);

If $\Phi(t) = \Phi_0 \cos \omega t$, then $\frac{\Delta \Phi}{\Delta t} = -\omega \Phi_0 \sin \omega t$ (for small $\Delta t$).
Sinusoidal $\textbf{Emf}$ Voltages

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Example 20.4

The magnetic field between the poles of an electromagnet has constant magnitude $B$. A circular coil of wire immersed in this magnetic field has $N$ turns and area $A$. An externally applied torque causes the coil to rotate with constant angular velocity $\omega$ about an axis perpendicular to the field.

Use Faraday’s “law” to find the $\text{emf}$ voltage induced in the coil.
Example 20.4 Strategy

The magnetic field does not vary, but the orientation of the coil does.

The number of field lines crossing through the coil depends on the angle that the field makes with the normal (the direction perpendicular to the coil).

The changing magnetic flux induces an emf voltage in the coil, according to Faraday’s “law”.
Example 20.4 Solution 1

Let us choose $t = 0$ to be an instant when the field is perpendicular to the coil. At this instant, $\mathbf{B}$ is parallel to the normal, so $\theta = 0$.

At a later time $t > 0$, the coil has rotated through an angle $\Delta \theta = \omega t$. Thus, the angle that the field makes with the normal as a function of $t$ is

$$\theta = \omega t$$

The flux through the coil is

$$\Phi = B A \cos \theta = B A \cos \omega t$$
Example 20.4 Solution 2

\[
\frac{\Delta \Phi}{\Delta t} = \omega \Phi_0 \cos \omega t
\]

\[
\Phi_0 = BA
\]

\[
\varepsilon = -N \frac{\Delta \Phi}{\Delta t} = \omega NBA \sin \omega t
\]
Application: Ground Fault Interrupter

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Application: Moving Coil Microphone

Moving coil

Sound waves

Diaphragm

Stationary magnet

Induced current
20.4 Lenz’s “Law”

The directions of the induced \textit{emf} voltages and currents caused by a changing magnetic flux can be determined using \textit{Lenz’s “law”}.

**Lenz’s “Law”**

The direction of the induced current in a loop always opposes the \textit{change} in magnetic flux that induces the current.

Note that induced \textit{emf} voltages and currents do not necessarily oppose the magnetic field or the magnetic flux; they oppose the \textit{change} in the magnetic flux.
Example 20.5

Verify the emf voltages and currents calculated in Example 20.1 using Faraday’s and Lenz’s “laws”—that is, find the directions and magnitudes of the emf voltages and currents by looking at the changing magnetic flux through the loop.
Example 20.5 Strategy

To apply Faraday’s “law”, look for the reason why the flux is changing.

In Example 20.1, a loop moves to the right at constant velocity into, through, and then out of a region of magnetic field.

The magnitude and direction of the magnetic field within the region are not changing, nor is the area of the loop. What does change is the portion of that area that is immersed in the region of magnetic field.
Example 20.5 Solution 1

At positions 1, 3, and 5, the flux is not changing even though the loop is moving. In each case, a small displacement of the loop causes no flux change. The flux is zero at positions 1 and 5, and nonzero but constant at position 3. For these three positions, the induced emf voltage is zero and so is the current.
Example 20.5 Solution 2

If the loop were *at rest* at position 2, the magnetic flux would be constant. However, since the loop is moving into the region of field, the area of the loop through which magnetic field lines cross is increasing. Thus, the flux is increasing.
Example 20.5 Solution 3

According to Lenz’s “law”, the direction of the induced current opposes the change in flux. Since the field is into the page, and the flux is increasing, the induced current flows in the direction that produces a magnetic field out of the page. By the right-hand rule, the current is counterclockwise.
Example 20.5 Solution 4

At position 2, a length $x$ of the loop is in the region of magnetic field. The area of the loop that is immersed in the field is $Lx$.

$$\Phi_B = BA = BLx$$

$$\frac{\Delta \Phi_B}{\Delta t} = BL \frac{\Delta x}{\Delta t} = BLv$$

$$|\mathcal{E}| = BLv$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$
Example 20.5 Solution 5

At position 4, the flux is decreasing as the loop leaves the region of magnetic field. Once again, let a length $x$ of the loop be immersed in the field.

$$\Phi_B = BLx$$

$$|\mathcal{E}| = \left| \frac{\Delta \Phi_B}{\Delta t} \right| = BL \left| \frac{\Delta x}{\Delta t} \right| = BLv$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$

Flux is decreasing. To oppose a decrease, the induced current makes a magnetic field in the same direction as the external field —into the page. Then the current must be clockwise.
Example 20.6

A circular loop of wire moves toward a bar magnet at constant velocity. The loop passes around the magnet and continues away from it on the other side.

Use Lenz’s “law” to find the direction of the current in the loop at positions 1 and 2.
Example 20.6 Strategy

The magnetic flux through the loop is changing because the loop moves from weaker to stronger field (at position 1), and vice versa (at position 2).

We can specify current directions as counterclockwise or clockwise as viewed from the left (with the loop moving away).
Example 20.6 Solution 1

At position 1, the magnetic field lines enter the magnet at the south pole, so the field lines cross the loop from left to right. Since the loop is moving closer to the magnet, the field is getting stronger.
Example 20.6 Solution 2

The number of field lines crossing the loop increases. The flux is therefore increasing. To oppose the increase, the current makes a magnetic field to the left. The right-hand rule gives the current direction to be counterclockwise as viewed from the left.
Example 20.6 Solution 3

At position 2, the field lines still cross the loop from left to right, but now the field is getting weaker. The current must flow in the opposite direction—clockwise as viewed from the left.
20.5 Back Emf in a Motor

A generator and a motor are essentially the same device. Is there an induced emf voltage in the coil (or windings) of a motor?

There must be, according to Faraday’s “law”, since the magnetic flux through the coil changes as the coil rotates.

By Lenz’s “law”, this induced emf voltage—called a back emf—opposes the flow of current in the coil, since it is the current that makes the coil rotate and thus causes the flux change.

The magnitude of the back emf depends on the rate of change of the flux, so the back emf increases as the rotational speed of the coil increases.
**Back Emf in a Motor 2**

Assume that this motor has many coils (also called windings) at all different angles so that the torques, emf voltages, and currents are all constant.

When the external emf voltage is first applied, there is no back emf because the windings are not rotating.

Then the current has a maximum value $I = \frac{\mathcal{E}_{\text{ext}}}{R}$. The faster the motor turns, the greater the back emf, and the smaller the current: $I = (\mathcal{E}_{\text{ext}} - \mathcal{E}_{\text{back}})/R$. 
**Back Emf in a Motor 3**

You may have noticed that when a large motor—as in a refrigerator or washing machine—first starts up, the room lights dim a bit.

The motor draws a large current when it starts up because there is no *back emf*. The voltage drop across the wiring in the walls is proportional to the current flowing in them, so the voltage across lightbulbs and other loads on the circuit is reduced, causing a momentary “brownout.”

As the motor comes up to speed, the current drawn is much smaller, so the brownout ends.
If a motor is overloaded, so that it turns slowly or not at all, the current through the windings is large.

Motors are designed to withstand such a large current only momentarily, as they start up; if the current is sustained at too high a level, the motor “burns out”—the windings heat up enough to do damage to the motor.
20.6 Transformers

The figures show two simple transformers. In each, two separate strands of insulated wire are wound around an iron core. The magnetic field lines are guided through the iron, so the two coils enclose the same magnetic field lines.
Transformers 2

An alternating voltage is applied to the primary coil; the ac current in the primary causes a changing magnetic flux through the secondary coil.

If the primary coil has $N_1$ turns, an emf voltage $\mathcal{E}_1$ is induced in the primary coil according to Faraday’s “law”:

$$\mathcal{E}_1 = -N_1 \frac{\Delta \Phi_b}{\Delta t}$$

Here $\Delta \Phi_b/\Delta t$ is the rate of change of the flux through each turn of the primary. Ignoring resistance in the coil and other energy losses, the induced emf voltage is equal to the ac voltage applied to the primary.
Transformers 3

If the secondary coil has $N_2$ turns, then the emf voltage induced in the secondary coil is

$$E_2 = -N_2 \frac{\Delta \Phi_B}{\Delta t}$$

At any instant, the flux through each turn of the secondary is equal to the flux through each turn of the primary, so $\Delta \Phi_B/\Delta t$ is the same quantity in both of these:

$$E_1 = -N_1 \frac{\Delta \Phi_B}{\Delta t} \quad E_2 = -N_2 \frac{\Delta \Phi_B}{\Delta t}$$

Eliminating $\Delta \Phi_B/\Delta t$ from the two equations, we find the ratio of the two emf voltages to be

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
The ratio \( N_2 / N_1 \) is called the **turns ratio**.

A transformer is often called a *step-up* or a *step-down* transformer, depending on whether the secondary *emf* voltage is larger or smaller than the *emf* voltage applied to the primary.

The same transformer may often be used as a step-up or step-down transformer depending on which coil is used as the primary.
Transformers: Current Ratio

In an *ideal transformer*, power losses in the transformer itself are negligible.

Most transformers are very efficient, so ignoring power loss is usually reasonable.

Then the rate at which energy is supplied to the primary is equal to the rate at which energy is supplied by the secondary ($P_1 = P_2$). Since power equals voltage times current, the ratio of the currents is the inverse of the ratio of the emf voltages:

$$\frac{I_2}{I_1} = \frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{N_1}{N_2}$$
Example 20.7

A transformer inside the charger for a cell phone has 500 turns in the primary coil. It supplies an emf voltage of amplitude 6.8 V when plugged into the usual sinusoidal household emf voltage of amplitude 170 V.

(a) How many turns does the secondary coil have?

(b) If the current drawn by the cell phone has amplitude 1.50 A, what is the amplitude of the current in the primary?
Example 20.7 Strategy

The ratio of the emf voltages is the same as the turns ratio.

We know the two emf voltages and the number of turns in the primary, so we can find the number of turns in the secondary.

To find the current in the primary, we assume an ideal transformer. Then the currents in the two are inversely proportional to the emf voltages.
Example 20.7 Solution

(a) \[ \frac{\%_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \]

\[ N_2 = \frac{\mathcal{E}_2}{\mathcal{E}_1} N_1 = \frac{6.8V}{170V} \times 500 = 20 \text{ turns} \]

(b) \[ \frac{I_1}{I_2} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \]

\[ I_1 = \frac{\mathcal{E}_2}{\mathcal{E}_1} I_2 = \frac{6.8V}{170V} \times 1.50A = 0.060A \]
20.7 Eddy Currents

Whenever a conductor is subjected to a changing magnetic flux, the induced emf voltage causes currents to flow.

In a solid conductor, induced currents flow simultaneously along many different paths. These are called eddy currents because of their resemblance to swirling eddies of current in air or in the rapids of a river.

Though the pattern of current flow is complicated, we can still use Lenz’s “law” to get a general idea of the direction of the current flow (clockwise or counterclockwise).

We can also determine the qualitative effects of eddy current flow using energy conservation. Since they flow in a resistive medium, the eddy currents dissipate electric energy.
Example 20.8

A balance must have some damping mechanism. Without one, the balance arm would tend to oscillate for a long time before it settles down; determining the mass of an object would be a long, tedious process.

A typical device used to damp out the oscillations is shown in the figure.
A metal plate attached to the balance arm passes between the poles of a permanent magnet.

(a) Explain the damping effect in terms of energy conservation.

(b) Does the damping force depend on the speed of the plate?

**Strategy**

As portions of the metal plate move into or out of the magnetic field, the changing magnetic flux induces *emf* voltages. These induced *emf* voltages cause the flow of eddy currents. Lenz’s “law” determines the direction of the eddy currents.
Example 20.8 Solution 1

(a) As the plate moves between the magnet poles, parts of it move into the magnetic field while other parts move out of the field.

Due to the changing magnetic flux, induced emf voltages cause eddy currents to flow. The eddy currents dissipate energy; the energy must come from the kinetic energy of the balance arm, pan, and object on the pan.

As the currents flow, the kinetic energy of the balance decreases and it comes to rest much sooner than it would otherwise.
Example 20.8 Solution 2

(b) If the plate is moving faster, the flux is changing faster.

Faraday’s “law” says that the induced $\text{emf}$ voltages are proportional to the rate of change of the flux.

Larger induced $\text{emf}$ voltages cause larger currents to flow. The damping force is the magnetic force acting on the eddy currents.

Therefore, the damping force is larger.
20.8 Induced Electric Fields

When a conductor moves in a magnetic field, a motional emf voltage arises due to the magnetic force on the mobile charges.

Since the charges move along with the conductor, they have a nonzero average velocity. The magnetic force on these charges pushes them around the circuit if a complete circuit exists.

What causes the induced emf voltage in a stationary conductor in a changing magnetic field?

Now the conductor is at rest, and the mobile charges have an average velocity of zero. The average magnetic force on them is then zero, so it cannot be the magnetic force that pushes the charges around the circuit.
Induced Electric Fields 2

An **induced electric field**, created by the changing magnetic field, acts on the mobile charge in the conductor, pushing it around the circuit. The same force “law” \( \mathbf{F} = q \mathbf{E} \) applies to induced electric fields as to any other electric field.

The induced **emf** voltage around a loop is the work done per unit charge on a charged particle that moves around the loop.

Thus, an induced electric field does nonzero work on a charge that moves around a closed path, starting and ending at the same point. In other words, the induced electric field is nonconservative.

The work done by the induced \( \mathbf{E} \) field *cannot* be described as the charge times the potential difference.
## Conservative Versus Nonconservative E Fields

**Table 20.1** Comparison of Conservative and Nonconservative E Fields.

<table>
<thead>
<tr>
<th>Source</th>
<th>Conservative E Fields</th>
<th>Nonconservative (Induced) E Fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field lines</td>
<td>Charges</td>
<td>Changing B fields</td>
</tr>
<tr>
<td>Start on positive charges and end on negative charges</td>
<td>Closed loops</td>
<td></td>
</tr>
<tr>
<td>Can be described by an electric potential?</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Work done over a closed path</td>
<td>Always zero</td>
<td>Can be nonzero</td>
</tr>
</tbody>
</table>
Electromagnetic Fields

How can Faraday’s “law” give the induced emf voltage regardless of why the flux is changing—whether because of a changing magnetic field or because of a conductor moving in a magnetic field?

A conductor that is moving in one frame of reference is at rest in another frame of reference. As we will see in Chapter 26, Einstein’s theory of special relativity says that either reference frame is equally valid. In one frame, the induced emf voltage is due to the motion of the conductor; in the other, the induced emf voltage is due to a changing magnetic field.
The electric and magnetic fields are not really separate entities. They are intimately connected.

Though it is advantageous in many circumstances to think of them as distinct fields, a more accurate view is to think of them as two aspects of the **electromagnetic field**.

To use a loose analogy: a vector has different $x$- and $y$-components in different coordinate systems, but these components represent the same vector quantity.

In the same way, the electromagnetic field has electric and magnetic parts (analogous to vector components) that depend on the frame of reference.
Electromagnetic Fields

A purely electric field in one frame of reference has both electric and magnetic “components” in another reference frame.

You may notice a missing symmetry. If a changing $\mathbf{B}$ field is always accompanied by an induced $\mathbf{E}$ field, what about the other way around?

Does a changing electric field make an induced magnetic field?

The answer to this important question—central to our understanding of light as an electromagnetic wave—is yes (Chapter 22).
20.9 Inductance

The figure shows two coils of wire. A power supply with variable emf voltage causes current $I_1$ to flow in coil 1; the current produces magnetic field lines as shown.
Some of these field lines cross through the turns of coil 2. If we adjust the power supply so that $I_1$ changes, the flux through coil 2 changes and an induced emf voltage appears in coil 2.
Mutual Inductance

**Mutual inductance**—when a changing current in one device causes an induced emf voltage in another device—can occur between two circuit elements in the same circuit as well as between circuit elements in two different circuits.
Mutual Inductance

In either case, a changing current through one element induces an emf voltage in the other. The effect is truly mutual: a changing current in coil 2 induces an emf voltage in coil 1 as well.
Self-Inductance

American scientist Joseph Henry (1797–1878) was the first to wrap insulated wires around an iron core to make an electromagnet.

Henry was also the first to suggest that a changing current in a coil induces an emf voltage in the *same* coil—an effect called **self-inductance** (or **inductance** for short).
Inductors

When a coil, solenoid, toroid, or other circuit element is used in a circuit primarily for its self-inductance effects, it is called an inductor.

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Inductance Defined

The inductance $L$ of an inductor is defined as the constant of proportionality between the self-flux through the inductor and the current $I$ flowing through the inductor windings.

**Definition of inductance:**

$$N \Phi = LI$$

where the flux through each turn is $\Phi$ and the inductor has $N$ turns.

The SI unit for inductance is called the henry (symbol H).

$$1 \text{H} = 1 \frac{\text{Wb}}{\text{A}} = 1 \frac{\text{Wb/s}}{\text{A/s}} = 1 \frac{\text{V} \cdot \text{s}}{\text{A}}$$
Induced Emf Voltage in an Inductor

When the current in the inductor changes, the flux changes. $N$ and $L$ are constants, so $N \Delta \Phi = L \Delta I$.

Then, from Faraday’s “law”, the induced emf voltage in the inductor is.

$$E = -N \frac{\Delta \Phi}{\Delta t} = -L \frac{\Delta I}{\Delta t}$$

The induced emf voltage is proportional to the rate of change of the current.
Inductance of a Solenoid

The most common form of inductor is the solenoid. In Problem 51, the self-inductance $L$ of a long air core solenoid of $n$ turns per unit length, length $\ell$, and radius $r$ is found to be.

$$L = \mu_0 n^2 \pi r^2,$$

In terms of the total number of turns $N$, where $N = n\ell$, the inductance is.

$$L = \frac{\mu_0 N^2 \pi r^2}{\ell^2}$$
Inductors in Circuits

The behavior of an inductor in a circuit can be summarized as current *stabilizer*.
Inductors Store Energy 1

An inductor stores energy in a magnetic field, just as a capacitor stores energy in an electric field.

Suppose the current in an inductor increases at a constant rate from 0 to \( I \) in a time \( T \). We let lowercase \( i \) stand for the instantaneous current at some time \( t \) between 0 and \( T \), and let uppercase \( I \) stand for the final current.

The instantaneous rate at which energy accumulates in the inductor is:

\[
P = \mathcal{E} i
\]
Inductors Store Energy 2

Since current increases at a constant rate, the magnetic flux increases at a constant rate, so the induced $\text{emf}$ voltage is constant.

Also, since the current increases at a constant rate, the average current is $I_{av} = I/2$. Then the average rate at which energy accumulates is:

$$P_{av} = E_{av} = \frac{1}{2} E I$$

Using

$$E = -N \frac{\Delta \Phi}{\Delta t} = -L \frac{\Delta I}{\Delta t}$$

for the $\text{emf}$ voltage, the average power is

$$P_{av} = \frac{1}{2} L \frac{\Delta i}{\Delta t} I$$
Inductors Store Energy 3

\[ P_{av} = \frac{1}{2} L \frac{\Delta i}{\Delta t} I \]

and the total energy stored in the inductor is.

\[ U = P_{av} T = \frac{1}{2} \left( L \frac{\Delta i}{\Delta t} \right) IT \]
Magnetic energy stored in an inductor

Since the current changes at a constant rate, $\Delta i/\Delta t = I/T$. The total energy stored in the inductor is.

**Magnetic energy stored in an inductor:**

$$U = \frac{1}{2} LI^2$$

Although to simplify the calculation we assumed that the current was increased from zero at a constant rate, the equation for the energy stored in an inductor depends only on the current $I$ and not on how the current reached that value.
Magnetic Energy Density 1

We can use the inductor to find the magnetic energy density in a magnetic field.

Consider a solenoid so long that we can ignore the magnetic energy stored in the field outside it. The inductance is.

\[ L = \mu_0 n^2 \pi r^2 \ell \]

where \( n \) is the number of turns per unit length, \( \ell \) is the length of the solenoid, and \( r \) is its radius.
Magnetic Energy Density 2

The energy stored in the inductor when a current I flows is

\[ U = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 n^2 \pi r^2 \ell I^2 \]

The volume of space inside the solenoid is the length times the cross-sectional area:

\[ \text{volume} = \pi r^2 \ell \]

Then the magnetic energy density—energy per unit volume—is

\[ u_B = \frac{U}{\pi r^2 \ell} = \frac{1}{2} \mu_0 n^2 I^2 \]
Magnetic Energy Density 3

To express the energy density in terms of the magnetic field strength, recall that \( B = \mu_0 nI \) inside a long solenoid. Therefore,

**Magnetic energy density:**

\[
u_B = \frac{1}{2 \mu_0} B^2
\]

The equation is valid for more than the interior of an air core solenoid; it gives the energy density for *any* magnetic field except for the field inside a ferromagnet.
Example 20.9

The main magnet in an MRI machine is a large solenoid whose windings are superconducting wire kept cold by liquid helium. The solenoid is 2.0 m long and 0.60 m in diameter.

During normal operation, the current through the windings is 120 A and the magnetic field strength is 1.4 T.
Example 20.9 (2)

(a) How much energy is stored in the magnetic field during normal operation?

(b) During an accidental quench, part of the coil becomes a normal conductor instead of a superconductor. The energy stored in the magnet is then rapidly dissipated. How many moles of liquid helium can be boiled by the energy stored in the magnet? (The latent heat of vaporization of helium is 82.9 J/mol.) At 20°C and 1 atm, what volume would this amount of helium occupy?

(c) After necessary repairs, the magnet is restarted by connecting the solenoid to a 18-V power supply. How long does it take for the current to reach 120 A?
Example 20.9 Strategy

(a) The energy stored can be found from the inductance and the current, but the problem gives the magnetic field rather than the inductance, so an easier approach starts by calculating the magnetic energy density.

The energy stored is then the energy density (energy per unit volume) times the volume of the solenoid.
Example 20.9 Strategy

Using the energy found in part (a) along with the latent heat, we can calculate the number of moles of helium that boil. Then the ideal gas “law” relates the number of moles to the volume the helium occupies.

The superconducting windings of the solenoid have zero electrical resistance, so we can treat the solenoid as an ideal inductor. When connected to a power supply, Kirchhoff’s loop rule requires that the induced emf voltage in the solenoid be equal to the emf voltage of the power supply.
Example 20.9 Solution 1

(a)

\[ V = \pi r^2 \ell \]

\[ U = u_b \pi r^2 \ell = \frac{1}{2\mu_0} B^2 \pi r^2 \ell \]

\[ U = \frac{1}{2\left(4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A}\right)(1.4 \text{T})^2 \pi (0.30 \text{m})^2 (2.0 \text{m})} \]

\[ = 0.441\text{MJ} \rightarrow 0.44\text{MJ} \]
Example 20.9 Solution 2

(b) 

\[ n = \frac{U}{L_v} = \frac{0.441 \text{MJ}}{82.9 \text{J/mol}} = 5300 \text{mol} \]

\[ V = n \frac{RT}{P} = \frac{U}{L_v} \frac{RT}{P} \]

\[ = \frac{0.441 \times 10^6 \text{J} \times 8.31 \frac{j}{\text{K mol}} \times 293 \text{K}}{82.9 \frac{j}{\text{mol}} \times 101.3 \times 10^3 \text{Pa}} = 130 \text{m}^3 \]

Asphyxiation is a serious danger when an accidental quench occurs.
Example 20.9 Solution 3

(c)

\[ I_f = 120 \text{ A} \]

\[ U = \frac{1}{2} LI_f^2 \Rightarrow L = \frac{2U}{I_f^2} \]

\[ |\mathcal{E}| = L \frac{\Delta I}{\Delta t} \Rightarrow \Delta t = \frac{L \Delta I}{|\mathcal{E}|} = \frac{2U \Delta I}{I_f^2 |\mathcal{E}|} \]

\[ \Delta I = I_f \]

\[ \Delta t = \frac{2 \left(0.441 \times 10^6 \text{ J}\right) \left(120\text{ A}\right)}{(120\text{ A})^2 (18\text{ V})} = 408\text{ s} = 6.8\text{ min} \]
20.10 LR Circuits

To get an idea of how inductors behave in circuits, let’s first study them in dc circuits—that is, in circuits with batteries or other constant-voltage power supplies. Consider the LR circuit in the figure.

The inductor is assumed to be ideal: its windings have negligible resistance. At $t = 0$, the switch $S$ is closed. What is the subsequent current in the circuit?
LR Circuits 2

The current through the inductor just before the switch is closed is zero. As the switch is closed, the current is initially zero.

An instantaneous change in current through an inductor would mean an instantaneous change in its stored energy, since $U \propto I^2$. An instantaneous change in energy means that energy is supplied in zero time.

Since nothing can supply infinite power…

Current through an inductor must always change continuously, never instantaneously.
LR Circuits 3

The initial current is zero, so there is no voltage drop across the resistor. The magnitude of the induced emf voltage in the inductor \((E_L)\) is initially equal to the battery’s emf voltage \((E_b)\). Therefore, the current is rising at an initial rate given by:

\[
\frac{\Delta I}{\Delta t} = \frac{E_b}{L}
\]
As current builds up, the voltage drop across the resistor increases. Then the induced emf voltage in the inductor ($\mathcal{E}_L$) gets smaller so that.

\[ \mathcal{E}_b - \mathcal{E}_L - IR = 0 \]
\[ \mathcal{E}_b = \mathcal{E}_L + IR \]
LR Circuits 5

Since the voltage across an *ideal* inductor is the induced emf voltage, we can substitute $E_L = L(\Delta I/\Delta t)$:

$$E_b = L \frac{\Delta I}{\Delta t} + IR$$

The battery emf voltage is constant. Thus, as the current increases, the voltage drop across the resistor gets larger and the induced emf voltage in the inductor gets smaller. Therefore, the rate at which the current increases gets smaller.
After a very long time, the current reaches a stable value. Since the current is no longer changing, there is no voltage drop across the inductor, so $E_b = I_f R$ or:

$$I_f = \frac{E_b}{R}$$
The current as a function of time $I(t)$ is:

$$I(t) = I_0 \left(1 - e^{-t/\tau}\right)$$

**Time constant, LR circuit:**

$$\tau = \frac{L}{R}$$

The induced emf voltage as a function of time is.

$$\mathcal{E}_L(t) = \mathcal{E}_b - IR = \mathcal{E}_b - \frac{\mathcal{E}_b}{R} \left(1 - e^{-t/\tau}\right)R = \mathcal{E}_b \ e^{-t/\tau}$$
LR Circuits 8

The $LR$ circuit in which the current is initially zero is analogous to the charging $RC$ circuit.

In both cases, the device starts with no stored energy and gains energy after the switch is closed.

In charging a capacitor, the $\text{charge}$ eventually reaches a nonzero equilibrium value, while for the inductor the $\text{current}$ reaches a nonzero equilibrium value.
LR Circuits 9

What about an LR circuit analogous to the discharging RC circuit? That is, once a steady current is flowing through an inductor, and energy is stored in the inductor, how can we stop the current and reclaim the stored energy?

Simply opening the switch would not be a good way to do it. The attempt to suddenly stop the current would induce a huge emf voltage in the inductor.

Most likely, sparks would complete the circuit across the open switch, allowing the current to die out more gradually. (Sparking generally isn’t good for the health of the switch.)
A Better Way to Stop the Current

\[ I_0 = \frac{\mathcal{E}_b}{R_i} \]

\[ I(t) = I_0 e^{-t/\tau} \]

\[ \tau = \frac{L}{R_2} \]
LR and RC Circuits Compared

**CONNECTION:**
This summary shows that RC and LR circuits are closely analogous.

<table>
<thead>
<tr>
<th></th>
<th>Capacitor</th>
<th>Inductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage is proportional to</td>
<td>Charge(^3)</td>
<td>Rate of change of current</td>
</tr>
<tr>
<td>Can change discontinuously</td>
<td>Current</td>
<td>Voltage</td>
</tr>
<tr>
<td>Cannot change discontinuously</td>
<td>Voltage</td>
<td>Current</td>
</tr>
<tr>
<td>Energy stored ((U)) is proportional to</td>
<td>(V^2)</td>
<td>(F)</td>
</tr>
<tr>
<td>When (V = 0) and (I \neq 0)</td>
<td>(U = 0)</td>
<td>(U = ) maximum</td>
</tr>
<tr>
<td>When (I = 0) and (V \neq 0)</td>
<td>(U = ) maximum</td>
<td>(U = 0)</td>
</tr>
<tr>
<td>Energy stored ((U)) is proportional to</td>
<td>(E^2)</td>
<td>(B^2)</td>
</tr>
<tr>
<td>Time constant =</td>
<td>(RC)</td>
<td>(L/R)</td>
</tr>
<tr>
<td>“Charging” circuit</td>
<td>(I(t) \propto e^{-\frac{t}{RC}})</td>
<td>(I(t) \propto (1 - e^{-\frac{t}{RC}}))</td>
</tr>
<tr>
<td></td>
<td>(V_C(t) \propto (1 - e^{-\frac{t}{RC}}))</td>
<td>(V_C(t) = %_t(t) \propto e^{\frac{t}{RC}})</td>
</tr>
<tr>
<td>“Discharging” circuit</td>
<td>(I(t) \propto e^{-\frac{t}{RC}})</td>
<td>(I(\Phi) \propto e^{\frac{t}{L/R}} \propto e^{\frac{t}{RC}})</td>
</tr>
<tr>
<td></td>
<td>(V_C(t) \propto e^{\frac{t}{RC}})</td>
<td></td>
</tr>
</tbody>
</table>
Example 20.10

A large electromagnet has an inductance $L = 15 \text{ H}$. The resistance of the windings is $R = 8.2 \Omega$. Treat the electromagnet as an ideal inductor in series with a resistor.

When a switch is closed, a 24-V dc power supply is connected to the electromagnet.

(a) What is the ultimate current through the windings of the electromagnet?

(b) How long after closing the switch does it take for the current to reach 99.0% of its final value?
Example 20.10 Strategy

When the current reaches its final value, there is no induced emf voltage.

The ideal inductor in the figure therefore has no potential difference across it. Then the entire voltage of the power source is across the resistor.

The current follows an exponential curve as it builds to its final value.

When it is at 99.0% of its final value, it has 1.0% left to go.
Example 20.10 Solution 1

(a) After the switch has been closed for many time constants, the current reaches a steady value. When the current is no longer changing, there is no induced emf voltage.

\[ E_b = E_L + IR \]

when \( E_L = 0, \ i = \frac{E_b}{R} = \frac{24V}{8.2\Omega} = 2.9A \)
Example 20.10 Solution 2

(b) The factor $e^{-t/\tau}$ represents the fraction of the current yet to build up. When the current reaches 99.0% of its final value,

$$1 - e^{-t/\tau} = 0.990 \text{ or } e^{-t/\tau} = 0.010$$

$$\ln\left(e^{-t/\tau}\right) = -t / \tau = \ln 0.010 = -4.61$$

$$t = -\tau \ln 0.010 = -\frac{L}{R} \ln 0.010 = -\frac{15\text{H}}{8.2\Omega} \times (-4.61) = 8.4\text{s}$$

It takes 8.4 s for the current to build up to 99.0% of its final value.