Chapter 17: Electric Potential

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17.1 Electric Potential Energy

Electric potential energy is the energy stored in an electric field.
Electric Potential Energy and Work

For both gravitational and electric potential energy, the change in potential energy when objects move around is equal in magnitude but opposite in sign to the work done by the field:

\[ \Delta U = W_{\text{field}} \]

The amount of energy \( W_{\text{field}} \) is taken from stored potential energy. The field dips into its “potential energy bank account” and gives the energy to the object, so the potential energy decreases when the force does positive work.
Some of the many similarities between gravitational and electric potential energy include:

• In both cases, the potential energy depends on only the positions of various objects, not on the path they took to get to those positions.

• Only changes in potential energy are physically significant, so we are free to assign the potential energy to be zero at any one convenient point.

• For two point particles, we usually choose \( U = 0 \) when the particles are infinitely far apart.
Both the gravitational and electrical forces exerted by one point particle on another are inversely proportional to the square of the distance between them ($F \propto 1/r^2$). As a result, the gravitational and electric potential energies have the same distance dependence ($U \propto 1/r$, with $U = 0$ at $r = \infty$).

The gravitational force and the gravitational potential energy for a pair of point particles are proportional to the product of the masses of the particles:

$$F = \frac{G m_1 m_2}{r^2}$$

$$U_g = -\frac{G m_1 m_2}{r^2} \quad (U_g = 0 \text{ at } r = \infty)$$
The electric force and the electric potential energy for a pair of point particles are proportional to the product of the charges of the particles:

$$F = \frac{k |q_1| |q_2|}{r^2}$$

$$U_E = \frac{kq_1q_2}{r} \quad (U_E = 0 \text{ at } r = \infty)$$
Electric Potential Energy Graphed

(a) Gravitational attraction
(b) Electrical attraction ($q_1q_2 < 0$)
(c) Electrical repulsion ($q_1q_2 > 0$)
Example 17.1

In a thunderstorm, charge is separated through a complicated mechanism that is ultimately powered by the Sun.

A simplified model of the charge in a thundercloud represents the positive charge accumulated at the top and the negative charge at the bottom as a pair of point charges.
Example 17.1 (2)

(a) What is the electric potential energy of the pair of point charges, assuming that \( U = 0 \) when the two charges are infinitely far apart?

(b) Explain the sign of the potential energy in light of the fact that \textit{positive} work must be done by external forces in the thundercloud to \textit{separate} the charges.
Example 17.1 Strategy

(a) The electric potential energy for a pair of point charges is given by.

\[ U_E = \frac{kq_1q_2}{r} \]

\((U_E = 0 \text{ at } r = \infty)\)

where \(U = 0\) at infinite separation is assumed. The algebraic signs of the charges are included when finding the potential energy.

(b) The work done by an external force to separate the charges is equal to the change in the electric potential energy as the charges are moved apart by forces acting within the thundercloud.
Example 17.1 Solution 1

(a)

\[ U_E = \frac{kq_1q_2}{r} \]

\[
U_E = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2} \frac{(+50 C)(-20 C)}{8000 m}
\]

\[ = -1 \cdot 10^9 N \cdot m = -1 \cdot 10^9 J \]
Example 17.1 Solution 2

(b) Recall that we chose \( U = 0 \) at infinite separation.

Negative potential energy therefore means that, *if the two point charges started infinitely far apart*, their electric potential energy would decrease as they are brought together—in the absence of other forces they would “fall” spontaneously toward one another.

However, in the thundercloud, the unlike charges *start close together* and are moved *farther apart* by an external force; the external agent must do *positive* work to increase the potential energy and move the charges *apart*.

Initially, when the charges are close together, the potential energy is *less than* \(-1 \times 10^9\) J; the *change* in potential energy as the charges are moved apart is *positive*. 
Example: two people near one another

(a)

\[ q_1 = q_2 \approx 1 \text{nC} \]
\[ r \approx 2 \text{m} \]

\[ U_E = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2} \left(10^{-9} \text{C}\right)^2 \frac{2 \text{m}}{2 \text{m}} \]

\[ = 5 \text{nJ} \]
Potential Energy due to Several Point Charges

Imagine you bring in charge $q_1$ first. This requires no work, since there is no charge to oppose (or help). When you bring in the second charge, $q_2$, the energy is:

$$U_{12} = \frac{k q_1 q_2}{r_{12}}$$

If I now bring in a third charge, $q_3$, there are TWO new interactions:

$$U_{13} = \frac{k q_1 q_3}{r_{13}} \quad U_{23} = \frac{k q_2 q_3}{r_{23}}$$

The potential energy is the negative of the work done by the electric field as the three charges are put into their positions, starting from infinite separation.
Potential Energy due to Several Point Charges

For three point charges, there are three pairs, so the TOTAL potential energy is.

\[ U_E = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]

!! If I now bring in a fourth charge, \( q_4 \), how many additional terms are there? Write out the new equation for four charges. HINT: How many “pairs” are there?
Example 17.2

Find the electric potential energy for the array of charges shown in the figure. Charge $q_1 = +4.0 \, \mu C$ is located at (0.0, 0.0) m; charge $q_2 = +2.0 \, \mu C$ is located at (3.0, 4.0) m; and charge $q_3 = -3.0 \, \mu C$ is located at (3.0, 0.0) m.
Example 17.2 Strategy

With three charges, there are three pairs to include in the potential energy sum.

The charges are given; we need only find the distance between each pair.

Subscripts are useful to identify the three distances; $r_{12}$, for example, means the distance between $q_1$ and $q_2$. 
Example 17.2 Solution

\[ r_{12} = \sqrt{3.0^2 + 4.0^2} \, \text{m} = \sqrt{25} \, \text{m} = 5.0 \, \text{m} \]

\[ U_E = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]

\[ U_E = 8.99 \times 10^9 \, \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \left[ \frac{(+4.0)(+2.0)}{5.0} + \frac{(+4.0)(-3.0)}{3.0} + \frac{(+2.0)(-3.0)}{4.0} \right] \]

\[ \times 10^{-12} \, \frac{\text{C}^2}{\text{m}} = -0.035 \, \text{J} \]
17.2 Electric Potential

Just as the electric field is defined as the electric force per unit charge, the electric potential $V$ is defined as the electric potential energy per unit charge.

Electric potential is often shortened to potential. It is also informally called “voltage”.

$$1 \text{ V} = 1 \text{ J/C}$$
17.2 Electric Potential

Note that positive charges “fall” toward low potential (or low voltage) and negative charges “fall” toward high potential (or high voltage). But BOTH “fall” to lower potential energy.
Potentials do not have direction in space; they are added just as any other scalar.

Potentials can be either positive or negative and so must be added with their algebraic signs.

If the potential at a point due to a collection of fixed charges is $V$, then when a charge $q$ is placed at that point, the electric potential energy is.

$$U_E = qV$$
Potential Difference

When a point charge $q$ moves from point $A$ to point $B$, it moves through a *potential difference*.

$$\Delta V = V_f - V_i = V_B - V_A$$

The potential difference is the change in electric potential energy per unit charge:

$$\Delta U_E = q \Delta V$$
In a region where the electric field is zero, the potential is constant.

Recall that a force points in the direction of \textbf{DECREASING} potential energy. In the same way:

$$\vec{E} \text{ points in the direction of decreasing } V.$$
Example 17.3

A battery-powered lantern is switched on for 5.0 min. During this time, electrons with total charge $-8.0 \times 10^2$ C flow through the lamp; 9600 J of electric potential energy is converted to light and heat.

Through what potential difference do the electrons move?
Example 17.3 Strategy

The equation.

\[ \Delta U_E = q \Delta V \]

relates the change in electric potential energy to the potential difference.

We could apply the equation to a single electron, but since all of the electrons move through the same potential difference, we can let \( q \) be the total charge of the electrons and \( \Delta U_E \) be the total change in electric potential energy.
Example 17.3 Solution

\[ \Delta V = \frac{\Delta U_E}{q} = \frac{-9600 \text{ J}}{-8.0 \times 10^2 \text{ C}} = +12 \text{ V} \]
Potential due to a Point Charge

If \( q \) is in the vicinity of one other point charge \( Q \), the electric potential energy is.

\[
U = \frac{kQq}{r}
\]

Therefore, the electric potential at a distance \( r \) from a point charge \( Q \) is.

\[
V = \frac{kQ}{r} \quad (V \text{ is } 0 \text{ at } r = \infty)
\]
Superposition of Potentials

The potential at a point $P$ due to $N$ point charges is the sum of the potentials due to each charge:

$$V = \sum V_i = \sum \frac{kQ_i}{r_i} \text{ for } i = 1, 2, 3, \ldots, N$$

where $r_i$ is the distance from the $i^{\text{th}}$ point charge $Q_i$ to point $P$. 
Example 17.4

Charge $Q_1 = +4.0 \, \mu\text{C}$ is located at (0.0, 3.0) cm; charge $Q_2 = +2.0 \, \mu\text{C}$ is located at (1.0, 0.0) cm; and charge $Q_3 = -3.0 \, \mu\text{C}$ is located at (2.0, 2.0) cm.

(a) Find the electric potential at point $A(x = 0.0, y = 1.0 \, \text{cm})$ due to the three charges.

(b) A point charge $q = -5.0 \, \text{nC}$ moves from a great distance to point $A$. What is the change in electric potential energy?
Example 17.4 Strategy

The potential at $A$ is the sum of the potentials due to each point charge.

The first step is to find the distance from each charge to point $A$. We call these distances $r_1$, $r_2$, and $r_3$ to avoid using the wrong one by mistake.

Then we add the potentials due to each of the three charges at $A$. 

Example 17.4 Solution 1

(a)

\[ r_1 = 2.0 \text{ cm} \]

\[ r_2 = \sqrt{2.0 \text{ cm}} = 1.414 \text{ cm} \]

\[ r_3 = \sqrt{1.0^2 + 2.0^2 \text{ cm}} = \sqrt{5.0} \text{ cm} = 2.236 \text{ cm} \]

\[ V = k \sum \frac{Q_i}{r_i} \]
Example 17.4 Solution 2

(a) continued.

\[ V_A = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2} \times \left( \frac{+4.0 \times 10^{-6} C}{0.020 \ m} + \frac{+2.0 \times 10^{-6} C}{0.01414 \ m} + \frac{-3.0 \times 10^{-6} C}{0.02236 \ m} \right) \]

\[ = +1.863 \times 10^6 \text{ V} \]
Example 17.4 Solution 3

(b) \[ \Delta U_E = q(V_A - 0) = (-5.0 \times 10^{-9} \text{ C}) \times (+1.863 \times 10^6 \text{ J/C} - 0) \]
\[ = -9.3 \times 10^{-3} \text{ J} \]
Example 17.5

Four equal positive point charges $q$ are fixed at the corners of a square of side $s$.

(a) Is the electric field zero at the center of the square?
(b) Is the potential zero at the center of the square?
Example 17.5 Strategy and Solution 1

The electric field at the center is the vector sum of the fields due to each of the point charges. The figure shows the field vectors at the center of the square due to each charge.

Each of these vectors has the same magnitude since the center is equidistant from each corner and the four charges are the same. From symmetry, the vector sum of the electric fields is zero.
Example 17.5 Strategy and Solution 2

(b) Since potential is a scalar rather than a vector, the potential at the center of the square is the \textit{scalar} sum of the potentials due to each charge.

These potentials are all equal since the distances and charges are the same. Each is positive since \( q > 0 \). The total potential at the center of the square is.

\[
V = A \frac{kq}{r} \quad r = s\sqrt{2}
\]
In Section 16.4, we saw that the field outside a charged conducting sphere is the same as if all of the charge were concentrated into a point charge located at the center of the sphere.

As a result, the electric potential due to a conducting sphere is similar to the potential due to a point charge.
The electric field inside the conducting sphere (from $r = 0$ to $r = R$) is zero.

The magnitude of the electric field is greatest at the surface of the conductor and then drops off as $1/ r^2$.

Outside the sphere, the electric field is the same as for a charge $Q$ located at $r = 0$. 
The potential is chosen to be zero for \( r = \infty \). The electric field outside the sphere (\( r \geq R \)) is the same as the field at a distance \( r \) from a point charge \( Q \).

Therefore, for any point at a distance \( r \geq R \) from the center of the sphere, the potential is the same as the potential at a distance \( r \) from a point charge \( Q \):

\[
V = \frac{kQ}{r} \quad (r \geq R)
\]

At the surface of the sphere, the potential is.

\[
V = \frac{kQ}{R}
\]
Potential due to a Spherical Conductor

Since the electric field inside the cavity is zero, no work would be done by the electric field if a test charge were moved around within the cavity.

Therefore, the potential anywhere inside the sphere is the same as the potential at the surface of the sphere.

Thus, for $r < R$, the potential is not the same as for a point charge. (The magnitude of the potential due to a point charge continues to increase as $r \to 0$.)
EXAMPLE: Potential of a person

Treat the person as a sphere of \( R \approx 1 \text{ m} \) (no, a person is not a sphere, so this will not give us 1% accuracy – but it is an EASY way to get an order of magnitude answer)

The charge (as always) is about 1 \( nC \).

So the voltage will be of the order of:

\[
V \approx \frac{kQ}{R} = \frac{9 \cdot 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \cdot 10^{-9} \text{ C}}{1 \text{ m}} = 9 \text{ V}
\]
Application: van de Graaff Generator

Conducting sphere

$E = 0$ inside

Comb to collect charge

Conveyor belt carrying electrons to collecting rods within sphere

Uncharged belt returning

Insulating cylinder

Motor to drive conveyor

Source of charge
Example 17.6

You wish to charge a van de Graaff to a potential of 240 kV. On a day with average humidity, an electric field of $8.0 \times 10^5$ N/C or greater ionizes air molecules, allowing charge to leak off the van de Graaff.

Find the minimum radius of the conducting sphere under these conditions.
Example 17.6 Strategy

We set the potential of a conducting sphere equal to $V_{\text{max}} = 240 \text{ kV}$ and require the electric field strength just outside the sphere to be less than $E_{\text{max}} = 8.0 \times 10^5 \text{ N/C}$. Since both $E$ and $V$ depend on the charge on the sphere and its radius, we should be able to eliminate the charge and solve for the radius.
Example 17.6 Solution

Comparing the two expressions, we see that $E = V / R$ just outside the sphere. Now let $V = V_{\text{max}}$ and require $E < E_{\text{max}}$:

$$E = \frac{V_{\text{max}}}{R} < E_{\text{max}}$$

$$R > \frac{V_{\text{max}}}{E_{\text{max}}} = \frac{2.4 \times 10^5 \text{ V}}{8.0 \times 10^5 \text{ N/C}}$$

$$R > 0.30 \text{ m}$$

The minimum radius is 30 cm.
Potential Differences in Biological Systems

In general, the inside and outside of a biological cell are *not* at the same potential.

The potential difference across a cell membrane is due to different concentrations of ions in the fluids inside and outside the cell.

These potential differences are particularly noteworthy in nerve and muscle cells.
Application: Transmission of Nerve Impulses
Application: Electrocardiographs

An electrocardiograph (ECG) measures the potential difference between points on the chest as a function of time.

The depolarization and polarization of the cells in the heart causes potential differences that can be measured using electrodes connected to the skin.
The potential difference measured by the electrodes is amplified and recorded on a chart recorder or a computer.
A field line sketch is a useful visual representation of the electric field.

To represent the electric potential, we can create something analogous to a contour map.

An **equipotential surface** has the same potential at every point on the surface.
Equipotential Surfaces

The idea is similar to the lines of constant elevation on a topographic map, which show where the elevation is the same.
Equipotential Surfaces and Field Lines

\[ \Delta U = -W_E = -F_E \Delta x \cos \theta \]
\[ q \Delta V = -q E \Delta x \cos \theta \]
\[ \Delta V = -E \Delta x \cos \theta \]

You can tell from this that:
1) the electric field is perpendicular to an equipotential surface.
2) the electric field points toward lower voltage
3) the electric field points in the direction of the MAXIMUM voltage DROP (gradient)

Rearrange to get:

\[ E \cos \theta = -\frac{\Delta V}{\Delta x} \]

Electric field is equal to the GRADIENT of the voltage.
EXAMPLE: What is the field created by a 9V battery?

Electric field is equal to the GRADIENT of the voltage.

\[
E = - \frac{\Delta V}{\Delta x} = - \frac{9 \text{V} - 0 \text{V}}{5 \text{mm}} = -1.8 \text{ kV/m}
\]
The Relationship Between Electric Field and Potential

Properties of a charge $q$ at a point in space due to its interaction with charges at other points

- Electric force ($\vec{F}_E = q \vec{E}$)

Properties of a point in space due to charges at other points

- Electric field ($\vec{E}$)

Vector quantities

- Is the negative gradient of the

Scalar quantities

- Electric potential energy ($U_E = qV$)

- Electric potential ($V$)

Per unit charge =

Access the text alternative for these images
Equipotential Surfaces

If equipotential surfaces are drawn such that the potential difference between adjacent surfaces is constant, then the surfaces are closer together where the field is stronger.

The electric field always points in the direction of maximum potential decrease.
Equipotential Surfaces near a Positive Point Charge.

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Example 17.7

Sketch some equipotential surfaces for two point charges $+Q$ and $-Q$.

Strategy and Solution.

One way to draw a set of equipotential surfaces is to first draw the field lines.

Then we construct the equipotential surfaces by sketching lines that are perpendicular to the field lines at all points.
Example 17.7 Solution

What is the voltage of this plane?
Potential in a Uniform Electric Field

In a uniform electric field, the field lines are equally spaced parallel lines.

Since equipotential surfaces are perpendicular to field lines, the equipotential surfaces are a set of parallel planes.

The potential decreases from one plane to the next in the direction of $\mathbf{E}$.

Since the spacing of equipotential planes depends on the magnitude of $\mathbf{E}$, in a uniform field planes at equal potential increments are equal distances apart.
Quantitative Relationship Between Electric Field and Potential

To find a quantitative relationship between the field strength and the spacing of the equipotential planes, imagine moving a point charge $+q$ a distance $d$ in the direction of an electric field of magnitude $E$.

The work done by the electric field is.

$$W_E = F_E d = qEd$$

The change in electric potential energy is.

$$\Delta U_E = -W_E = -qEd$$
Quantitative Relationship Between Electric Field and Potential

From the definition of potential, the potential change is.

\[ \Delta V = \frac{\Delta U}{q} = -Ed \]

The equation implies that the SI unit of the electric field (N/C) can also be written *volts per meter* (V/m):

\[ 1 \text{ N/C} = 1 \text{ V/m} \]

Where the field is strong, the equipotential surfaces are close together: with a large number of volts per meter, it doesn’t take many meters to change the potential a given number of volts.
Potential Inside a Conductor

In Section 16.6, we learned that $E = 0$ at every point inside a conductor in electrostatic equilibrium (when no charges are moving).

If the field is zero at every point, then the potential does not change as we move from one point to another. If there were potential differences within the conductor, then charges would move in response. Positive charge would be accelerated by the field toward regions of lower potential, and negative charge would be accelerated toward higher potential.

In electrostatic equilibrium, every point within a conducting material must be at the same potential.
17.4 Conservation of Energy for Moving Charges

When a charge moves from one position to another in an electric field, the change in electric potential energy must be accompanied by a change in other forms of energy so that the total energy is constant.

Energy conservation simplifies problem solving just as it does with gravitational or elastic potential energy.

If no other forces act on a point charge, then as it moves in an electric field, the sum of the kinetic and electric potential energy is constant:

\[ K_i + U_i = K_f + U_f = \text{constant} \]
Conservation of Energy for Moving Charges

Changes in gravitational potential energy are often negligible compared with changes in electric potential energy (when the gravitational force is much weaker than the electric force).
Example 17.8

In an electron gun, electrons are accelerated from the cathode toward the anode, which is at a potential higher than the cathode (see figure on next slide).

If the potential difference between the cathode and anode is 12 kV, at what speed do the electrons move as they reach the anode?

Assume that the initial kinetic energy of the electrons as they leave the cathode is negligible.
Example 17.8

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Example 17.8 Strategy

Using energy conservation, we set the sum of the initial kinetic and potential energies equal to the sum of the final kinetic and potential energies.

The initial kinetic energy is taken to be zero. Once we find the final kinetic energy, we can solve for the speed.

Known: \( K_i = 0; \Delta V = +12 \text{ kV} \)
Find: \( \nu \)
Example 17.8 Solution 1

\[ \Delta U = U_f - U_i = q\Delta V \]

\[ K_i + U_i = K_f + U_f \]

\[ K_f = K_i + (U_i - U_f) = K_i - \Delta U = 0 - q\Delta V \]

\[ K_f = \frac{1}{2} m\nu^2 \]

\[ \frac{1}{2} m\nu^2 = q\Delta V \]

\[ \nu = \sqrt{\frac{-2q\Delta V}{m}} \]
Example 17.8 Solution 2

\[ v = \sqrt{\frac{-2q\Delta V}{m}} \]

\[ q = -e = -1.602 \times 10^{-19} \text{ C} \]
\[ m = 9.109 \times 10^{-31} \text{ Kg} \]

\[ v = \sqrt{\frac{-2 \times (-1.602 \times 10^{-19} \text{ C}) \times (12,000 \text{ V})}{9.109 \times 10^{-31} \text{ kg}}} \]

\[ = 6.5 \times 10^7 \text{ m/s} \]
**Example: Photoelectric effect**

\[ K + U = \text{const} \]

\[ U = q \ E = q \ V \ d \]

\[ \frac{1}{2} m v_0^2 + qE d = 0 + 0 \]

\[ v_0 = \sqrt{\frac{2 qE d}{m}} = \sqrt{\frac{2 qV}{m}} \]

(same as before?)
A **capacitor** is a device that stores electric potential energy by storing separated positive and negative charges.

It consists of two conductors separated by either vacuum or an insulating material. Charge is separated, with positive charge put on one of the conductors and an equal amount of negative charge on the other conductor.

The arrows indicate a few of the many capacitors on a circuit board from a computer.
Field Lines in a Parallel Plate Capacitor

There is a potential difference between the two plates; the positive plate is at the higher potential. Between the plates (not too close to the edges), the field lines are straight, parallel, and uniformly spaced.
Capacitors

Work must be done to separate positive charge from negative charge, since there is an attractive force between the two.

The work done to separate the charge ends up as electric potential energy. An electric field arises between the two conductors, with field lines beginning on the conductor with positive charge and ending on the conductor with negative charge.

The stored potential energy is associated with this electric field. We can recover the stored energy—that is, convert it into some other form of energy—by letting the charges come together again.
Capacitors – analogy to water tower

Work must be done to raise water above the Earth, since there is an attractive force between the two.

The work done to raise the water ends up as gravitational potential energy.

We can recover the stored energy—that is, convert it into some other form of energy—by letting the water come down again.

People NEED water pressure, but mostly only in the morning and evening. An expensive pump that could do all that would be idle most of the day.

So we can have a very weak pump that takes all day to raise enough water to let everybody take a shower and do the dishes in the evening.
Parallel Plate Capacitors

The simplest form of capacitor is a parallel plate capacitor, consisting of two parallel metal plates, each of the same area \( A \), separated by a distance \( d \).

A charge \( +Q \) is put on one plate and a charge \( -Q \) on the other. For now, assume there is air between the plates.
Charging a Capacitor

One way to charge the plates is to connect the positive terminal of a battery to one and the negative terminal to the other.

The battery removes electrons from one plate, leaving it positively charged, and puts them on the other plate, leaving it with an equal magnitude of negative charge.

In order to do this, the battery has to do work—some of the battery’s chemical energy is converted into electric potential energy.
In general, the field between two such plates does not have to be uniform. However, if the plates are close together, then a good approximation is to say that the charge is evenly spread on the inner surfaces of the plates and none is found on the outer surfaces.

The plates in a real capacitor are almost always close enough that this approximation is valid.
Surface Charge Density

With charge evenly spread on the inner surfaces, a uniform electric field exists between the two plates.

We can neglect the non-uniformity of the field near the edges as long as the plates are close together. The electric field lines start on positive charges and end on negative charges.

If charge of magnitude $Q$ is evenly spread over each plate with surface of area $A$, then the *surface charge density* (the charge per unit area) is denoted by $\sigma$, the Greek letter sigma:

$$\sigma = \frac{Q}{A}$$
Electric Field just outside a Conductor

Gauss’s law (Section 16.7) can be used to show that the magnitude of the electric field just outside a conductor is

\[
E = 4\pi k \sigma = \sigma / \varepsilon_0
\]

\[
\varepsilon_0 = 1 / (4\pi k) = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)
\]
Potential Difference Between the Plates

Since the field is uniform, the magnitude of the potential difference between the plates is.

\[ \Delta V = Ed \]

The field is proportional to the charge and the potential difference is proportional to the field; therefore, the charge is proportional to the potential difference.

That turns out to be true for any capacitor, not just a parallel plate capacitor.
Definition of Capacitance

\[ Q = C \Delta V \]

where \( Q \) is the magnitude of the charge on each plate and \( \Delta V \) is the magnitude of the potential difference between the plates.

The constant of proportionality \( C \) is called the capacitance.

The SI units of capacitance are coulombs per volt, which is called the farad (symbol F).
Capacitance of Parallel Plate Capacitor 1

We can now find the capacitance of a parallel plate capacitor. The electric field is.

\[ E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \]

where \( A \) is the inner surface area of each plate. If the plates are a distance \( d \) apart, then the magnitude of the potential difference is.

\[ \Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \]

By rearranging, this can be rewritten in the form

\[ Q = \text{constant} \times \Delta V: \]

\[ Q = \frac{\varepsilon_0 A}{d} \Delta V \]
Comparing with the definition of capacitance, the capacitance of a parallel plate capacitor is.

**Capacitance of parallel plate capacitor:**

\[ C = \frac{\varepsilon_0 A}{d} = \frac{A}{4\pi kd} \]

!! Put in the units of \( \varepsilon_0 \) and show that you get a farad (i.e., a coulomb / volt) for the units.
Example 17.9

In one kind of computer keyboard, each key is attached to one plate of a parallel plate capacitor; the other plate is fixed in position.
Example 17.9

The capacitor is maintained at a constant potential difference of 5.0 V by an external circuit.

When the key is pressed down, the top plate moves closer to the bottom plate, changing the capacitance and causing charge to flow through the circuit.

If each plate is a square of side 6.0 mm and the plate separation changes from 4.0 mm to 1.2 mm when a key is pressed, how much charge flows through the circuit?

Does the charge on the capacitor increase or decrease? Assume that there is air between the plates instead of a flexible insulator.
Example 17.9 Strategy

Since we are given the area and separation of the plates, we can find the capacitance from:

\[ C = \frac{\varepsilon_0 A}{d} = \frac{A}{4\pi kd} \]

The charge is then found from the product of the capacitance and the potential difference across the plates: \( Q = C \Delta V \).
Example 17.9 Solution

\[ C = \frac{A}{4\pi kd} \]

\[ Q_f - Q_i = C_f \Delta V - C_i \Delta V \]
\[ = \left( \frac{A}{4\pi kd_f} - \frac{A}{4\pi kd_i} \right) \Delta V = \frac{A\Delta V}{4\pi k} \left( \frac{1}{d_f} - \frac{1}{d_i} \right) \]

\[ Q_f - Q_i = \frac{(0.0060 \text{ m})^2 \times 5.0 \text{ V}}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \times \left( \frac{1}{0.0012 \text{ m}} - \frac{1}{0.0040 \text{ m}} \right) \]
\[ = +9.3 \times 10^{-13} \text{ C} = +0.93 \text{ pC} \]

Since \( \Delta Q \) is positive, the magnitude of charge on the plates increases.
Example: Isolated sphere

Suppose a charge, Q, is pushed onto a sphere of radius, R. The Potential at the surface of the sphere is:

\[ \Delta V = \frac{kQ}{R} = \frac{Q}{4\pi R} \]

And so the capacitance of the sphere is:

\[ C \overset{\text{def}}{=} \frac{Q}{\Delta V} = \frac{Q}{kQ/R} = R/k = 4\pi \epsilon_0 R \]

Recall the parallel plate and you see that capacitance will always be \( \epsilon_0 \) times some kind of distance: \( C \sim \epsilon_0 \cdot 'distance' \)

!! Show that the unit of permittivity, \( \epsilon_0 \), can be written as F/m (= farad/meter)
Example: Charging while idling

Remember that we estimated that in a 10 minute drive, your car builds up a charge of 15 C, and I commented that it was a large overestimate. Here’s why I think that:

The capacitance of the car will be VERY ROUGHLY:

\[ C = \frac{R}{k} \sim \frac{2m}{9 \cdot 10^9 N \cdot m^2 / C^2} = 2 \cdot 10^{-10} C^2 / J = 0.2 nF \]

And so, for the car to hold a charge of 15C, it would be charged up to a voltage of:

\[ \Delta V = \frac{Q}{C} \sim \frac{15 C}{20 \cdot 10^{-9} F} = 0.75 \cdot 10^9 V = 0.75 GV \]
Example: Charging while idling

The maximum electric field in dry air is about 3 MV/m (at that point, air begins to conduct – it’s MUCH smaller for humid or polluted air.

!! Show that for a charged sphere, the electric field at the surface is related to the voltage at the surface by: \( E_{\text{MAX}} = \Delta V_{\text{MAX}}/R \)

If I use this, the maximum voltage of the car is:

\[
\Delta V_{\text{MAX}} = E_{\text{MAX}} R \sim (3 \text{ MV/m})(2 \text{ m}) = 6 \text{ MV}
\]

And so the maximum charge would be:

\[
Q_{\text{MAX}} = C \Delta V_{\text{MAX}} = \left(2 \cdot 10^{-10} \text{ C}^2/\text{J}\right)\left(6 \cdot 10^6 \text{ J/C}\right) = 12 \cdot 10^{-4} \text{ C} = 1.2 \text{ mC}
\]

(And probably lower still for humid or polluted air)
Application: Condenser Microphone

Fixed plate forms a capacitor with the diaphragm.

Moving plate (diaphragm) vibrates in response to sound wave.

Battery maintains a constant potential difference between the plates.

Processing circuit converts current into a varying output voltage.

Access the text alternative for these images
There is a problem inherent in trying to store a large charge in a capacitor. To store a large charge without making the potential difference excessively large, we need a large capacitance.

Capacitance is inversely proportional to the spacing $d$ between the plates. One problem with making the spacing small is that the air between the plates of the capacitor breaks down at an electric field of about 3000 V/mm with dry air (less for humid air).

The breakdown allows a spark to jump across the gap so the stored charge is lost.
One way to overcome this difficulty is to put a better insulator than air between the plates.

Some insulating materials, which are also called dielectrics, can withstand electric fields larger than those that cause air to break down and act as a conductor rather than as an insulator.

Another advantage of placing a dielectric between the plates is that the capacitance itself is increased.

(And the dielectric keeps the plates from touching!)
Parallel Plate Capacitor with Dielectric

For a parallel plate capacitor in which a dielectric fills the entire space between the plates, the capacitance is.

Capacitance of parallel plate capacitor with dielectric:

$$C = \kappa \frac{\varepsilon_0 A}{d} = \kappa \frac{A}{4\pi kd}$$

The effect of the dielectric is to increase the capacitance by a factor $\kappa$ (Greek letter kappa), which is called the dielectric constant.
Dielectric Strength

The dielectric constant depends on the insulating material used.

The **dielectric strength** is the electric field strength at which **dielectric breakdown** occurs and the material becomes a conductor.

Since $\Delta V = Ed$ for a uniform field, the dielectric strength determines the maximum potential difference that can be applied across a capacitor per meter of plate spacing.
Selected Dielectric Constants and Strengths

**Table 17.1** Dielectric Constants and Dielectric Strengths for Materials at 20°C (in Order of Increasing Dielectric Constant).

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength (kV/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1 (exact)</td>
<td>—</td>
</tr>
<tr>
<td>Air (dry, 1 atm)</td>
<td>1.00054</td>
<td>3</td>
</tr>
<tr>
<td>Paraffined paper</td>
<td>2.0–3.5</td>
<td>40–60</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Rubber (vulcanized)</td>
<td>3.0–4.0</td>
<td>16–50</td>
</tr>
<tr>
<td>Paper (bond)</td>
<td>3.0</td>
<td>8</td>
</tr>
<tr>
<td>Mica</td>
<td>4.5–8.0</td>
<td>150–220</td>
</tr>
</tbody>
</table>

Do not confuse dielectric constant and dielectric strength; they are not related.
Polarization in a Dielectric

What is happening microscopically to a dielectric between the plates of a capacitor?

Recall that polarization is a separation of the charge in an atom or molecule. The atom or molecule remains neutral, but the center of positive charge no longer coincides with the center of negative charge.
Polarization in a Dielectric

The charges on the capacitor plates induce a polarization of the dielectric.

The polarization occurs throughout the material, so the positive charge is slightly displaced relative to the negative charge.
Throughout the bulk of the dielectric, there are still equal amounts of positive and negative charge.

The net effect of the polarization of the dielectric is a layer of positive charge on one face and negative charge on the other. Each conducting plate faces a layer of opposing charge.
Dielectric Strength

The layer of opposing charge induced on the surface of the dielectric helps attract more charge to the conducting plate, for the same potential difference, than would be there without the dielectric.

Since capacitance is charge per unit potential difference, the capacitance must have increased.

The dielectric constant of a material is a measure of the ease with which the insulating material can be polarized.
Polarization in a Dielectric

Start with a charge, \( Q \), on the capacitor (and isolate the capacitor so \( Q \) is fixed).

The induced charge on the faces of the dielectric create a polarization field, \( \mathbf{E}_{\text{POL}} \), that opposes the initial field, \( \mathbf{E}_0 \).

The net electric field will be smaller: \( \mathbf{E} = \mathbf{E}_0 - \mathbf{E}_{\text{POL}} \). And so, for the amount of charge, \( Q \), the voltage will be smaller and so the capacitance will be larger.

The dielectric constant, \( K \), is the proportional increase in capacity.
Polarization in a Dielectric

The problem with this idea is that you almost NEVER have an isolated capacitor. It’s USUALLY in a circuit, at a fixed voltage, NOT a fixed charge.

!! Your mission, should you choose to accept it, is to explain how this model STILL works even if the VOLTAGE is constant instead of the charge. (HINT: The figure is still correct, but note that the voltage is the same with or without the dielectric. And so the net electric field is the same. For this to be true, what can you say about the charge on the capacitor?)
Dielectric Constant

Suppose a dielectric is immersed in an external electric field $E_0$. The definition of the dielectric constant is the ratio of the electric field in vacuum $E_0$ to the electric field $E$ inside the dielectric material:

Definition of dielectric constant: $\kappa = \frac{E_0}{E}$  

(and so:) $\kappa = \frac{\Delta V_0}{\Delta V}$

The electric field inside the dielectric ($E$) is.

$E = \frac{E_0}{\kappa}$

$\Delta V = \frac{\Delta V_0}{\kappa}$
Example 17.10

A parallel plate capacitor has plates of area 1.00 m$^2$ and spacing of 0.500 mm. The insulator has dielectric constant 4.9 and dielectric strength 18 kV/mm.

(a) What is the capacitance?

(b) What is the maximum charge that can be stored on this capacitor?
Example 17.10 Solution

(a) 

\[ C = \kappa \frac{A}{4\pi kd} \]

\[ = 4.9 \times \frac{1.00 \text{ m}^2}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times 5.00 \times 10^{-4} \text{ m}} \]

\[ = 8.67 \times 10^{-8} \text{ F} = 86.7 \text{ nF} \]

(b) 

\[ \Delta V = 18 \text{ kV/mm} \times 0.500 \text{ mm} = 9.0 \text{ kV} \]

\[ Q = C \Delta V = 8.67 \times 10^{-8} \text{ F} \times 9.0 \times 10^3 \text{ V} = 7.8 \times 10^{-4} \text{ C} \]
Example 17.11

A neuron can be modeled as a parallel plate capacitor, where the membrane serves as the dielectric and the oppositely charged ions are the charges on the “plates”.

Find the capacitance of a neuron and the number of ions (assumed to be singly charged) required to establish a potential difference of 85 mV. Assume that the membrane has a dielectric constant of $\kappa = 3.0$, a thickness of 10.0 nm, and an area of $1.0 \times 10^{-10} \text{ m}^2$. 
Example 17.11 Strategy

Since we know $\kappa$, $A$, and $d$, we can find the capacitance.

Then, from the potential difference and the capacitance, we can find the magnitude of charge $Q$ on each side of the membrane.

A singly charged ion has a charge of magnitude $e$, so $Q/e$ is the number of ions on each side.
Example 17.11 Solution

\[ C = \kappa \frac{A}{4\pi kd} \]

\[ C = 3.0 \times \frac{1.0 \times 10^{-10} \text{ m}^2}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times 10.0 \times 10^{-9} \text{ m}} \]

\[ = 2.66 \times 10^{-13} \text{ F} = 0.27 \text{ pF} \]

\[ Q = C \Delta V = 2.66 \times 10^{-13} \text{ F} \times 0.085 \text{ V} \]

\[ = 2.26 \times 10^{-14} \text{ C} = 0.023 \text{ pC} \]

\[ \text{number of ions} = \frac{2.26 \times 10^{-14} \text{ C}}{1.602 \times 10^{-19} \text{ C} / \text{ion}} = 1.4 \times 10^5 \text{ ions} \]
Application: Thunderclouds and Lightning

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The energy stored in the capacitor can be found by summing the work done by the battery to separate the charge. (NOTE that the energy extracted from the battery is: $Q \Delta V$)
17.7 Energy Stored in a Capacitor

Suppose we look at this process at some instant of time when one plate has charge $+q_i$, the other has charge $-q_i$, and the potential difference between the plates is $\Delta V_i$.

If $\Delta q_i$ is small,

\[
\Delta V_i = \frac{q_i}{C}
\]

\[
\Delta U_i = \Delta q_i \times \Delta V_i
\]

\[
U = \sum \Delta U_i = \sum \Delta q_i \times \Delta V_i
\]
17.7 Energy Stored in a Capacitor

\[ U = \text{area of triangle} = \frac{1}{2} \text{ (base } \times \text{ height)} \]

\[ U = \frac{1}{2} Q \Delta V \]

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Example 17.12

Fibrillation is a chaotic pattern of heart activity that is ineffective at pumping blood and is therefore life-threatening.

A device known as a *defibrillator* is used to shock the heart back to a normal beat pattern. The defibrillator discharges a capacitor through paddles on the skin, so that some of the charge flows through the heart.

(a) If an 11.0-μF capacitor is charged to 6.00 kV and then discharged through paddles into a patient’s body, how much energy is stored in the capacitor?

(b) How much charge flows through the patient’s body if the capacitor discharges completely?
Example 17.12 Strategy

There are three equivalent expressions for energy stored in a capacitor.

Since the capacitor is completely discharged, all of the charge initially on the capacitor flows through the patient’s body.
Example 17.12 Solution

(a) 
\[ U = \frac{1}{2} Q \Delta V = \frac{1}{2} (C \Delta V) \times \Delta V = \frac{1}{2} C (\Delta V)^2 \]

\[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \times 11.0 \times 10^{-6} \text{ F} \times (6.00 \times 10^3 \text{ V})^2 = 198 \text{ J} \]

(b) 
\[ Q = C \Delta V = 11.0 \times 10^{-6} \text{ F} \times 6.00 \times 10^3 \text{ V} = 0.0660 \text{ C} \]
Example: Charging while idling

How much energy in your gasoline is wasted by charging up your car?

The energy stored by charging up your car must come from someplace, and the only SOURCE of energy in your car is the gasoline. The energy stored in the charging of your car is:

\[
\Delta U = \frac{1}{2} Q \Delta V = (0.0012 \text{ C})(6 \cdot 10^6 \text{ V}) = 7200 \text{ J}
\]

Seems like a lot, but a gallon of gas has an energy content of about 127 MJ (https://en.wikipedia.org/wiki/Gasoline)

So this probably too-large amount is only 0.000004 % of the energy in your tank...
Potential energy is energy of interaction or field energy.

The energy stored in a capacitor is stored in the electric field between the plates. We can use the energy stored in a capacitor to calculate how much energy per unit volume is stored in an electric field $E$.

Why energy per unit volume? Two capacitors can have the same electric field but store different amounts of energy. The larger capacitor stores more energy, proportional to the volume of space between the plates.
Energy Stored in an Electric Field

Another way to think of it is that I can think of the energy as work done while moving charges around (as we’ve discussed) OR as the energy stored when I separate the charges and create an electric field.

Which is choose is, at the moment, a matter of convenience (and the answer has to be the same either way)

BUT when we get to magnetism, energy is ONLY conserved if SOME of the energy is in the electric and magnetic field. Then we HAVE to worry about this.
Energy Stored in an Electric Field

In a parallel plate capacitor, the energy stored is.

\[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \kappa \frac{A}{4\pi kd} (\Delta V)^2 \]

Assuming the field is uniform between the plates, the potential difference is.

\[ \Delta V \propto \frac{Ed}{\kappa} \]

\[ U = \frac{1}{2} \kappa \frac{A}{4\pi kd} \left( \frac{Ed}{\kappa} \right)^2 = \frac{1}{2} \kappa \frac{Ad}{4\pi k} E^2 \]

Then the energy density \( u \)—the electric potential energy per unit volume—is.

\[ u = \frac{U}{Ad} = \frac{1}{2} \kappa \frac{1}{4\pi k} E^2 = \frac{1}{2} \kappa \varepsilon_0 E^2 \]
EXAMPLE: Energy from the Sun

If I divide the “brightness” of sunlight (power/area) by the speed of light (m/s), we get the energy density (J/m³)

!! Show using the units that this is true.

\[ u = \frac{1360 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 4.5 \cdot 10^{-6} \text{ J/m}^3 \]

Using the expression for the energy density:

\[ E = \sqrt{\frac{2u}{\varepsilon_0}} = \sqrt{\frac{9 \cdot 10^{-6} \text{ J/m}^3}{8.85 \cdot 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2}} = 10^3 \frac{N}{C} = 10^3 \frac{V}{m} \]

!! Show that the units under the radical give both N/C and V/m
Chapter 17: Electric Potential

17.1 Electric Potential Energy.
17.2 Electric Potential.
17.3 The Relationship Between Electric Field and Potential.
17.4 Conservation of Energy for Moving Charges.
17.5 Capacitors.
17.6 Dielectrics.
17.7 Energy Stored in a Capacitor.
17.1 Electric Potential Energy

Electric potential energy is the energy stored in an electric field.
Electric Potential Energy and Work

For both gravitational and electric potential energy, the change in potential energy when objects move around is equal in magnitude but opposite in sign to the work done by the field:

\[ \Delta U = W_{\text{field}} \]

The amount of energy \( W_{\text{field}} \) is taken from stored potential energy. The field dips into its “potential energy bank account” and gives the energy to the object, so the potential energy decreases when the force does positive work.
Some of the many similarities between gravitational and electric potential energy include:

- In both cases, the potential energy depends on only the *positions* of various objects, not on the *path* they took to get to those positions.

- Only *changes* in potential energy are physically significant, so we are free to assign the potential energy to be zero at any *one* convenient point.

- For two point particles, we usually choose $U = 0$ when the particles are infinitely far apart.
Similarity to Gravitational Potential Energy

Both the gravitational and electrical forces exerted by one point particle on another are inversely proportional to the square of the distance between them \((F \propto 1/r^2)\). As a result, the gravitational and electric potential energies have the same distance dependence \((U \propto 1/r, \text{ with } U = 0 \text{ at } r = \infty)\).

The gravitational force and the gravitational potential energy for a pair of point particles are proportional to the product of the masses of the particles:

\[
F = \frac{Gm_1m_2}{r^2}
\]

\[
U_g = -\frac{Gm_1m_2}{r^2} \quad (U_g = 0 \text{ at } r = \infty)
\]
Similarity to Gravitational Potential Energy

The electric force and the electric potential energy for a pair of point particles are proportional to the product of the charges of the particles:

\[ F = \frac{k|q_1| |q_2|}{r^2} \]

\[ U_E = \frac{kq_1q_2}{r} \quad (U_E = 0 \text{ at } r = \infty) \]
Electric Potential Energy Graphed

(a) Gravitational attraction

(b) Electrical attraction ($q_1q_2 < 0$)

(c) Electrical repulsion ($q_1q_2 > 0$)
In a thunderstorm, charge is separated through a complicated mechanism that is ultimately powered by the Sun.

A simplified model of the charge in a thundercloud represents the positive charge accumulated at the top and the negative charge at the bottom as a pair of point charges.
Example 17.1 (2)

(a) What is the electric potential energy of the pair of point charges, assuming that $U = 0$ when the two charges are infinitely far apart?

(b) Explain the sign of the potential energy in light of the fact that positive work must be done by external forces in the thundercloud to separate the charges.
Example 17.1 Strategy

(a) The electric potential energy for a pair of point charges is given by.

\[
U_E = \frac{kq_1 q_2}{r} \quad (U_E = 0 \text{ at } r = \infty)
\]

where \( U = 0 \) at infinite separation is assumed. The algebraic signs of the charges are included when finding the potential energy.

(b) The work done by an external force to separate the charges is equal to the change in the electric potential energy as the charges are moved apart by forces acting within the thundercloud.
Example 17.1 Solution 1

(a)

\[ U_E = \frac{kq_1q_2}{r} \]

\[ U_E = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2} \frac{(+50 \, C)(-20 \, C)}{8000 \, m} \]

\[ = -1 \cdot 10^9 \, N \cdot m = -1 \cdot 10^9 \, J \]
Example 17.1 Solution 2

(b) Recall that we chose $U = 0$ at infinite separation.

Negative potential energy therefore means that, if the two point charges started infinitely far apart, their electric potential energy would decrease as they are brought together—in the absence of other forces they would “fall” spontaneously toward one another.

However, in the thundercloud, the unlike charges start close together and are moved farther apart by an external force; the external agent must do positive work to increase the potential energy and move the charges apart.

Initially, when the charges are close together, the potential energy is less than $-1 \times 10^9$ J; the change in potential energy as the charges are moved apart is positive.
Example: two people near one another

(a) \[ q_1 = q_2 \approx 1 \text{nC} \]
\[ r \approx 2 \text{m} \]

\[ U_E = 9 \times 10^9 \frac{N \cdot m^2}{C^2} \left(10^{-9} C\right)^2 \]
\[ = 5 \text{nJ} \]
Potential Energy due to Several Point Charges

Imagine you bring in charge \( q_1 \) first. This requires no work, since there is no charge to oppose (or help). When you bring in the second charge, \( q_2 \), the energy is:

\[
U_{12} = \frac{kq_1q_2}{r_{12}}
\]

If I now bring in a third charge, \( q_3 \), there are TWO new interactions:

\[
U_{13} = \frac{kq_1q_3}{r_{13}} \quad \text{and} \quad U_{23} = \frac{kq_2q_3}{r_{23}}
\]

The potential energy is the negative of the work done by the electric field as the three charges are put into their positions, starting from infinite separation.
Potential Energy due to Several Point Charges

For three point charges, there are three pairs, so the TOTAL potential energy is.

\[ U_E = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]

!! If I now bring in a fourth charge, \( q_4 \), how many additional terms are there? Write out the new equation for four charges. HINT: How many “pairs” are there?
Example 17.2

Find the electric potential energy for the array of charges shown in the figure. Charge $q_1 = +4.0 \, \mu\text{C}$ is located at $(0.0, 0.0)$ m; charge $q_2 = +2.0 \, \mu\text{C}$ is located at $(3.0, 4.0)$ m; and charge $q_3 = -3.0 \, \mu\text{C}$ is located at $(3.0, 0.0)$ m.
Example 17.2 Strategy

With three charges, there are three pairs to include in the potential energy sum.

The charges are given; we need only find the distance between each pair.

Subscripts are useful to identify the three distances; $r_{12}$, for example, means the distance between $q_1$ and $q_2$. 
Example 17.2 Solution

\[ r_{12} = \sqrt{3.0^2 + 4.0^2} = \sqrt{25} \text{ m} = 5.0 \text{ m} \]

\[ U_E = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]

\[ U_E = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \left[ \frac{(+4.0)(+2.0)}{5.0} + \frac{(+4.0)(-3.0)}{3.0} + \frac{(+2.0)(-3.0)}{4.0} \right] \times 10^{-12} \frac{\text{C}^2}{\text{m}} = -0.035 \text{ J} \]
17.2 Electric Potential

Just as the electric field is defined as the electric force per unit charge, the electric potential \( V \) is defined as the electric potential energy per unit charge.

Electric potential is often shortened to potential. It is also informally called “voltage”.

\[ 1 \, V = 1 \, J/C \]
17.2 Electric Potential

Note that positive charges “fall” toward low potential (or low voltage) and negative charges “fall” toward high potential (or high voltage). But BOTH “fall” to lower potential energy.
17.2 Electric Potential

Potentials do not have direction in space; they are added just as any other scalar.

Potentials can be either positive or negative and so must be added with their algebraic signs.

If the potential at a point due to a collection of fixed charges is $V$, then when a charge $q$ is placed at that point, the electric potential energy is.

$$U_E = qV$$
Potential Difference

When a point charge $q$ moves from point $A$ to point $B$, it moves through a potential difference.

$$\Delta V = V_f - V_i = V_B - V_A$$

The potential difference is the change in electric potential energy per unit charge:

$$\Delta U_E = q \Delta V$$
Electric Field and Potential Difference

Recall that a force points in the direction of DECREASING potential energy. In the same way:

\[ \mathbf{E} \] points in the direction of decreasing \( V \).

In a region where the electric field is zero, the potential is constant.
Example 17.3

A battery-powered lantern is switched on for 5.0 min. During this time, electrons with total charge $-8.0 \times 10^2$ C flow through the lamp; 9600 J of electric potential energy is converted to light and heat.

Through what potential difference do the electrons move?
Example 17.3 Strategy

The equation.

\[ \Delta U_E = q \Delta V \]

relates the change in electric potential energy to the potential difference.

We could apply the equation to a single electron, but since all of the electrons move through the same potential difference, we can let \( q \) be the total charge of the electrons and \( \Delta U_E \) be the total change in electric potential energy.
Example 17.3 Solution

\[ \Delta V = \frac{\Delta U_E}{q} = \frac{-9600 \text{ J}}{-8.0 \times 10^2 \text{ C}} = +12 \text{ V} \]
Potential due to a Point Charge

If $q$ is in the vicinity of one other point charge $Q$, the electric potential energy is.

$$U = \frac{kQq}{r}$$

Therefore, the electric potential at a distance $r$ from a point charge $Q$ is.

$$V = \frac{kQ}{r} \quad (V = 0 \text{ at } r = \infty)$$
**Superposition of Potentials**

The potential at a point $P$ due to $N$ point charges is the sum of the potentials due to each charge:

\[ V = \sum V_i = \sum \frac{kQ_i}{r_i} \text{ for } i = 1, 2, 3, \ldots, N \]

where $r_i$ is the distance from the $i^{th}$ point charge $Q_i$ to point $P$. 
Example 17.4

Charge $Q_1 = +4.0 \, \mu\text{C}$ is located at (0.0, 3.0) cm; charge $Q_2 = +2.0 \, \mu\text{C}$ is located at (1.0, 0.0) cm; and charge $Q_3 = -3.0 \, \mu\text{C}$ is located at (2.0, 2.0) cm.

(a) Find the electric potential at point $A(x = 0.0, y = 1.0$ cm) due to the three charges.

(b) A point charge $q = -5.0 \, \text{nC}$ moves from a great distance to point $A$. What is the change in electric potential energy?
Example 17.4 Strategy

The potential at $A$ is the sum of the potentials due to each point charge.

The first step is to find the distance from each charge to point $A$. We call these distances $r_1$, $r_2$, and $r_3$ to avoid using the wrong one by mistake.

Then we add the potentials due to each of the three charges at $A$. 
Example 17.4 Solution 1

(a)

\[ r_1 = 2.0 \text{ cm} \]
\[ r_2 = \sqrt{2.0} \text{ cm} = 1.414 \text{ cm} \]
\[ r_3 = \sqrt{1.0^2 + 2.0^2} \text{ cm} = \sqrt{5.0} \text{ cm} = 2.236 \text{ cm} \]

\[ V = k \sum \frac{Q_i}{r_i} \]
Example 17.4 Solution 2

(a) continued.

\[ V_A = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \left( \frac{+4.0 \times 10^{-6} \text{C}}{0.020 \text{ m}} + \frac{+2.0 \times 10^{-6} \text{C}}{0.01414 \text{ m}} + \frac{-3.0 \times 10^{-6} \text{C}}{0.02236 \text{ m}} \right) \]

\[ = +1.863 \times 10^6 \text{V} \]
Example 17.4 Solution 3

(b) \[ \Delta U_E = q \Delta V \]

\[ \Delta U_E = q(V_A - 0) = (-5.0 \times 10^{-9} \text{C}) \times (+1.863 \times 10^6 \text{J/C} - 0) \]

\[ = -9.3 \times 10^{-3} \text{J} \]
Example 17.5

Four equal positive point charges \( q \) are fixed at the corners of a square of side \( s \).

(a) Is the electric field zero at the center of the square?

(b) Is the potential zero at the center of the square?
Example 17.5 Strategy and Solution 1

The electric field at the center is the vector sum of the fields due to each of the point charges. The figure shows the field vectors at the center of the square due to each charge.

Each of these vectors has the same magnitude since the center is equidistant from each corner and the four charges are the same. From symmetry, the vector sum of the electric fields is zero.
Example 17.5 Strategy and Solution 2

(b) Since potential is a scalar rather than a vector, the potential at the center of the square is the *scalar* sum of the potentials due to each charge.

These potentials are all equal since the distances and charges are the same. Each is positive since $q > 0$. The total potential at the center of the square is:

$$V = 4 \frac{kq}{r}$$

$$r = s \sqrt{2}$$
Potential due to a Spherical Conductor

In Section 16.4, we saw that the field outside a charged conducting sphere is the same as if all of the charge were concentrated into a point charge located at the center of the sphere.

As a result, the electric potential due to a conducting sphere is similar to the potential due to a point charge.
Potential due to a Spherical Conductor

The electric field inside the conducting sphere (from $r = 0$ to $r = R$) is zero.

The magnitude of the electric field is greatest at the surface of the conductor and then drops off as $1/r^2$.

Outside the sphere, the electric field is the same as for a charge $Q$ located at $r = 0$. 
Potential due to a Spherical Conductor

The potential is chosen to be zero for $r = \infty$. The electric field outside the sphere ($r \geq R$) is the same as the field at a distance $r$ from a point charge $Q$.

Therefore, for any point at a distance $r \geq R$ from the center of the sphere, the potential is the same as the potential at a distance $r$ from a point charge $Q$:

$$V = \frac{kQ}{r} \quad (r \geq R)$$

At the surface of the sphere, the potential is:

$$V = \frac{kQ}{R}$$
Potential due to a Spherical Conductor

Since the electric field inside the cavity is zero, no work would be done by the electric field if a test charge were moved around within the cavity.

Therefore, the potential anywhere inside the sphere is the same as the potential at the surface of the sphere.

Thus, for \( r < R \), the potential is not the same as for a point charge. (The magnitude of the potential due to a point charge continues to increase as \( r \to 0 \).)
EXAMPLE: Potential of a person

Treat the person as a sphere of $R \approx 1\,m$ (no, a person is not a sphere, so this will not give us 1% accuracy – but it is an EASY way to get an order of magnitude answer)

The charge (as always) is about 1 $nC$.

So the voltage will be of the order of:

$$V \approx \frac{kQ}{R} = \frac{9 \cdot 10^9 N \cdot m^2/C^2 \cdot 10^{-9} C}{1\,m} = 9\,V$$
Application: van de Graaff Generator
Example 17.6

You wish to charge a van de Graaff to a potential of 240 kV. On a day with average humidity, an electric field of $8.0 \times 10^5$ N/C or greater ionizes air molecules, allowing charge to leak off the van de Graaff.

Find the minimum radius of the conducting sphere under these conditions.
Example 17.6 Strategy

We set the potential of a conducting sphere equal to $V_{\text{max}} = 240 \text{ kV}$ and require the electric field strength just outside the sphere to be less than $E_{\text{max}} = 8.0 \times 10^5 \text{ N/C}$.

Since both $E$ and $V$ depend on the charge on the sphere and its radius, we should be able to eliminate the charge and solve for the radius.
Example 17.6 Solution

\[ V = \frac{kQ}{R} \]

\[ E = \frac{kQ}{R^2} \]

Comparing the two expressions, we see that \( E = V / R \) just outside the sphere. Now let \( V = V_{\text{max}} \) and require \( E < E_{\text{max}} \):

\[ E = \frac{V_{\text{max}}}{R} < E_{\text{max}} \]

\[ R > \frac{V_{\text{max}}}{E_{\text{max}}} = \frac{2.4 \times 10^5 \text{ V}}{8 \times 10^5 \text{ N/C}} \]

\[ R > 0.30 \text{ m} \]

The minimum radius is 30 cm.
Potential Differences in Biological Systems

In general, the inside and outside of a biological cell are *not* at the same potential.

The potential difference across a cell membrane is due to different concentrations of ions in the fluids inside and outside the cell.

These potential differences are particularly noteworthy in nerve and muscle cells.
Application: Transmission of Nerve Impulses
Application: Electrocardiographs

An electrocardiograph (ECG) measures the potential difference between points on the chest as a function of time.

The depolarization and polarization of the cells in the heart causes potential differences that can be measured using electrodes connected to the skin.
The potential difference measured by the electrodes is amplified and recorded on a chart recorder or a computer.
17.3 The Relationship Between Electric Field and Potential

A field line sketch is a useful visual representation of the electric field.

To represent the electric potential, we can create something analogous to a contour map.

An **equipotential surface** has the same potential at every point on the surface.
Equipotential Surfaces.

The idea is similar to the lines of constant elevation on a topographic map, which show where the elevation is the same.
Equipotential Surfaces and Field Lines

\[ \Delta U = -W_E = -F_E \Delta x \cos \theta \]
\[ \frac{q \Delta V}{\Delta x} = -q E \Delta x \cos \theta \]
\[ \Delta V = -E \Delta x \cos \theta \]

You can tell from this that:
1) the electric field is perpendicular to an equipotential surface.
2) the electric field points toward lower voltage.
3) the electric field points in the direction of the MAXIMUM voltage DROP (gradient).

Rearrange to get:

\[ E \cos \theta = -\frac{\Delta V}{\Delta x} \]

Electric field is equal to the GRADIENT of the voltage.
EXAMPLE: What is the field created by a 9V battery?

Electric field is equal to the GRADIENT of the voltage.

\[
E = -\frac{\Delta V}{\Delta x} \\
= -\frac{9V - 0V}{5 mm} \\
= -1.8 kV/m
\]
The Relationship Between Electric Field and Potential

Vector quantities

Properties of a charge \( q \) at a point in space due to its interaction with charges at other points

- Electric force \( \mathbf{F}_E = q \mathbf{E} \)
- Per unit charge =

Is the negative gradient of the

- Electric field \( \mathbf{E} \)
- Is the negative gradient of the

Scalar quantities

Properties of a point in space due to charges at other points

- Electric potential energy \( U_E = qV \)
- Per unit charge =

- Electric potential \( V \)
Equipotential Surfaces

If equipotential surfaces are drawn such that the potential difference between adjacent surfaces is constant, then the surfaces are closer together where the field is stronger.

The electric field always points in the direction of maximum potential decrease.
Equipotential Surfaces near a Positive Point Charge.
Example 17.7

Sketch some equipotential surfaces for two point charges $+Q$ and $-Q$.

Strategy and Solution.

One way to draw a set of equipotential surfaces is to first draw the field lines.

Then we construct the equipotential surfaces by sketching lines that are perpendicular to the field lines at all points.
Example 17.7 Solution

What is the voltage of this plane?
Potential in a Uniform Electric Field

In a uniform electric field, the field lines are equally spaced parallel lines.

Since equipotential surfaces are perpendicular to field lines, the equipotential surfaces are a set of parallel planes.

The potential decreases from one plane to the next in the direction of \( \mathbf{E} \).

Since the spacing of equipotential planes depends on the magnitude of \( \mathbf{E} \), in a uniform field planes at equal potential increments are equal distances apart.
Quantitative Relationship Between Electric Field and Potential

To find a quantitative relationship between the field strength and the spacing of the equipotential planes, imagine moving a point charge +\(q\) a distance \(d\) in the direction of an electric field of magnitude \(E\).

The work done by the electric field is.

\[
W_E = F_E \cdot d = qEd
\]

The change in electric potential energy is.

\[
\Delta U_E = -W_E = -qEd
\]
Quantitative Relationship Between Electric Field and Potential

From the definition of potential, the potential change is.

\[ \Delta V = \frac{\Delta U}{q} = -Ed \]

The equation implies that the SI unit of the electric field (N/C) can also be written volts per meter (V/m):

\[ 1 \text{ N/C} = 1 \text{ V/m} \]

Where the field is strong, the equipotential surfaces are close together: with a large number of volts per meter, it doesn’t take many meters to change the potential a given number of volts.
Potential Inside a Conductor

In Section 16.6, we learned that $E = 0$ at every point inside a conductor in electrostatic equilibrium (when no charges are moving).

If the field is zero at every point, then the potential does not change as we move from one point to another. If there were potential differences within the conductor, then charges would move in response. Positive charge would be accelerated by the field toward regions of lower potential, and negative charge would be accelerated toward higher potential.

**In electrostatic equilibrium, every point within a conducting material must be at the same potential.**
17.4 Conservation of Energy for Moving Charges

When a charge moves from one position to another in an electric field, the change in electric potential energy must be accompanied by a change in other forms of energy so that the total energy is constant.

Energy conservation simplifies problem solving just as it does with gravitational or elastic potential energy.

If no other forces act on a point charge, then as it moves in an electric field, the sum of the kinetic and electric potential energy is constant:

\[ K_i + U_i = K_f + U_f = \text{constant} \]
Conservation of Energy for Moving Charges

Changes in gravitational potential energy are often negligible compared with changes in electric potential energy (when the gravitational force is much weaker than the electric force).
Example 17.8

In an electron gun, electrons are accelerated from the cathode toward the anode, which is at a potential higher than the cathode (see figure on next slide).

If the potential difference between the cathode and anode is 12 kV, at what speed do the electrons move as they reach the anode?

Assume that the initial kinetic energy of the electrons as they leave the cathode is negligible.
Example 17.82

[Diagram of a cathode-ray tube with labeled parts: Cathode, Anode, Electron beam, Uniform E field seen from side, Side view, Fluorescent screen, Conductive coating, Plates for horizontal deflection, Plates for vertical deflection, Electron gun, Heated filament (source of electrons).]

Access the text alternative for these images
Example 17.8 Strategy

Using energy conservation, we set the sum of the initial kinetic and potential energies equal to the sum of the final kinetic and potential energies.

The initial kinetic energy is taken to be zero. Once we find the final kinetic energy, we can solve for the speed.

Known: \( K_i = 0 \); \( \Delta V = +12 \text{ kV} \)
Find: \( v \)
Example 17.8 Solution 1

\[ \Delta U = U_f - U_i = q\Delta V \]

\[ K_i + U_i = K_f + U_f \]

\[ K_f = K_i + (U_i - U_f) = K_i - \Delta U = 0 - q\Delta V \]

\[ K_f = \frac{1}{2} mv^2 \]

\[ \frac{1}{2} mv^2 = -q\Delta V \]

\[ v = \sqrt{\frac{-2q\Delta V}{m}} \]
Example 17.8 Solution 2

\[ v = \sqrt{\frac{2q\Delta V}{m}} \]

\[ q = -e = -1.602 \times 10^{-19} \text{ C} \]
\[ m = 9.109 \times 10^{-31} \text{ Kg} \]

\[ v = \sqrt{\frac{-2 \times (-1.602 \times 10^{-19} \text{ C}) \times (12,000 \text{ V})}{9.109 \times 10^{-31} \text{ kg}}} \]
\[ = 6.5 \times 10^7 \text{ m/s} \]
Example: Photoelectric effect

\[ K + U = \text{const} \]
\[ U = q \vec{E} = q V d \]
\[ \frac{1}{2} m v_0^2 + qEd = 0 + 0 \]

\[ v_0 = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{2qV}{m}} \]

(same as before?)
A **capacitor** is a device that stores electric potential energy by storing separated positive and negative charges.

It consists of two conductors separated by either vacuum or an insulating material. Charge is separated, with positive charge put on one of the conductors and an equal amount of negative charge on the other conductor.

The arrows indicate a few of the many capacitors on a circuit board from a computer.
Field Lines in a Parallel Plate Capacitor

There is a potential difference between the two plates; the positive plate is at the higher potential. Between the plates (not too close to the edges), the field lines are straight, parallel, and uniformly spaced.
Work must be done to separate positive charge from negative charge, since there is an attractive force between the two.

The work done to separate the charge ends up as electric potential energy. An electric field arises between the two conductors, with field lines beginning on the conductor with positive charge and ending on the conductor with negative charge.

The stored potential energy is associated with this electric field. We can recover the stored energy—that is, convert it into some other form of energy—by letting the charges come together again.
Capacitors – analogy to water tower

Work must be done to raise water above the Earth, since there is an attractive force between the two.

The work done to raise the water ends up as gravitational potential energy.

We can recover the stored energy—that is, convert it into some other form of energy—by letting the water come down again.

People NEED water pressure, but mostly only in the morning and evening. An expensive pump that could do all that would be idle most of the day.

So we can have a very weak pump that takes all day to raise enough water to let everybody take a shower and do the dishes in the evening.
The simplest form of capacitor is a parallel plate capacitor, consisting of two parallel metal plates, each of the same area $A$, separated by a distance $d$. A charge $+Q$ is put on one plate and a charge $-Q$ on the other. For now, assume there is air between the plates.
Charging a Capacitor

One way to charge the plates is to connect the positive terminal of a battery to one and the negative terminal to the other.

The battery removes electrons from one plate, leaving it positively charged, and puts them on the other plate, leaving it with an equal magnitude of negative charge.

In order to do this, the battery has to do work—some of the battery’s chemical energy is converted into electric potential energy.
Parallel Plate Capacitors

In general, the field between two such plates does not have to be uniform. However, if the plates are close together, then a good approximation is to say that the charge is evenly spread on the inner surfaces of the plates and none is found on the outer surfaces.

The plates in a real capacitor are almost always close enough that this approximation is valid.
Surface Charge Density

With charge evenly spread on the inner surfaces, a uniform electric field exists between the two plates.

We can neglect the non-uniformity of the field near the edges as long as the plates are close together. The electric field lines start on positive charges and end on negative charges.

If charge of magnitude $Q$ is evenly spread over each plate with surface of area $A$, then the surface charge density (the charge per unit area) is denoted by $\sigma$, the Greek letter sigma:

$$\sigma = \frac{Q}{A}$$
Electric Field just outside a Conductor

Gauss’s law (Section 16.7) can be used to show that the magnitude of the electric field just outside a conductor is

\[ E = 4\pi k \sigma = \sigma / \varepsilon_0 \]

\[ \varepsilon_0 = 1 / (4\pi k) = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2) \]
Potential Difference Between the Plates

Since the field is uniform, the magnitude of the potential difference between the plates is.

\[ \Delta V = Ed \]

The field is proportional to the charge and the potential difference is proportional to the field; therefore, *the charge is proportional to the potential difference.*

That turns out to be true for any capacitor, not just a parallel plate capacitor.
**Definition of Capacitance**

\[ Q = C \Delta V \]

where \( Q \) is the magnitude of the charge on each plate and \( \Delta V \) is the magnitude of the potential *difference* between the plates.

The constant of proportionality \( C \) is called the **capacitance**.

The SI units of capacitance are coulombs per volt, which is called the *farad* (symbol F).
We can now find the capacitance of a parallel plate capacitor. The electric field is:

\[ E = \frac{\sigma}{\varepsilon_0} \frac{Q}{\varepsilon_0 A} \]

where \( A \) is the inner surface area of each plate. If the plates are a distance \( d \) apart, then the magnitude of the potential difference is:

\[ \Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \]

By rearranging, this can be rewritten in the form:

\[ Q = \frac{\varepsilon_0 A}{d} \Delta V \]
Capacitance of Parallel Plate Capacitor 2

Comparing with the definition of capacitance, the capacitance of a parallel plate capacitor is.

**Capacitance of parallel plate capacitor:**

\[
C = \frac{\varepsilon_0 A}{d} \times \frac{A}{4\pi kd}
\]

!! Put in the units of \(\varepsilon_0\) and show that you get a farad (i.e., a coulomb / volt) for the units.
Example 17.9

In one kind of computer keyboard, each key is attached to one plate of a parallel plate capacitor; the other plate is fixed in position.
Example 17.9

The capacitor is maintained at a constant potential difference of 5.0 V by an external circuit.

When the key is pressed down, the top plate moves closer to the bottom plate, changing the capacitance and causing charge to flow through the circuit.

If each plate is a square of side 6.0 mm and the plate separation changes from 4.0 mm to 1.2 mm when a key is pressed, how much charge flows through the circuit?

Does the charge on the capacitor increase or decrease? Assume that there is air between the plates instead of a flexible insulator.
Example 17.9 Strategy

Since we are given the area and separation of the plates, we can find the capacitance from.

\[ C = \varepsilon_0 \frac{A}{d} = \frac{A}{4\pi kd} \]

The charge is then found from the product of the capacitance and the potential difference across the plates: \( Q = C \Delta V \).
Example 17.9 Solution

\[ C = \frac{A}{4\pi kd} \]

\[ Q_f - Q_i = C_1 \Delta V - C_1 \Delta V \]
\[ = \left( \frac{A}{4\pi kd_f} - \frac{A}{4\pi kd_i} \right) \Delta V = \frac{A \Delta V}{4\pi k} \left( \frac{1}{d_f} - \frac{1}{d_i} \right) \]

\[ Q_f - Q_i = \frac{(0.0060 \text{ m})^2 \times 5.0 \text{ V}}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \times \left( \frac{1}{0.0012 \text{ m}} - \frac{1}{0.0040 \text{ m}} \right) \]
\[ = +9.3 \times 10^{-13} \text{ C} = +0.93 \text{ pC} \]

Since \( \Delta Q \) is positive, the magnitude of charge on the plates increases.
Example: Isolated sphere

Suppose a charge, Q, is pushed onto a sphere of radius, R. The potential at the surface of the sphere is:

$$\Delta V = \frac{kQ}{R} = \frac{Q}{4\pi R}$$

And so the capacitance of the sphere is:

$$C \equiv \frac{Q}{\Delta V} = \frac{Q}{kQ/R} = R/k = 4\pi \epsilon_0 R$$

Recall the parallel plate and you see that capacitance will always be $\epsilon_0$ times some kind of distance: $C \sim \epsilon_0 \text{'distance'}$

!! Show that the unit of permittivity, $\epsilon_0$, can be written as F/m (farad/meter)
Example: Charging while idling

Remember that we estimated that in a 10 minute drive, your car builds up a charge of 15 C, and I commented that it was a large overestimate. Here’s why I think that:

The capacitance of the car will be VERY ROUGHLY:

\[
C = \frac{R}{k} \approx \frac{2m}{9 \cdot 10^9 \text{N} \cdot \text{m}^2 / \text{C}^2} = 2 \cdot 10^{-10} \text{C}^2 / \text{J} = 0.2 \text{nF}
\]

And so, for the car to hold a charge of 15C, it would be charged up to a voltage of:

\[
\Delta V = \frac{Q}{C} \approx \frac{15 \text{C}}{2 \cdot 10^{-9} \text{F}} = 0.75 \cdot 10^9 \text{V} = 0.75 \text{GV}
\]
Example: Charging while idling

The maximum electric field in dry air is about 3 MV/m (at that point, air begins to conduct – it’s MUCH smaller for humid or polluted air.

!! Show that for a charged sphere, the electric field at the surface is related to the voltage at the surface by: \( E_{\text{MAX}} = \Delta V_{\text{MAX}} / R \)

If I use this, the maximum voltage of the car is:

\[ \Delta V_{\text{MAX}} = E_{\text{MAX}} \times R \approx (3 \text{ MV/m})(2 \text{ m}) = 6 \text{ MV} \]

And so the maximum charge would be:

\[
Q_{\text{MAX}} = C \Delta V_{\text{MAX}} = (2 \cdot 10^{-10} \text{ C}^2/\text{J}) (6 \cdot 10^6 \text{ J/C}) \\
= 12 \cdot 10^{-4} \text{ C} = 1.2 \text{ mC}
\]

(And probably lower still for humid or polluted air)
Application: Condenser Microphone

Fixed plate forms a capacitor with the diaphragm.

Moving plate (diaphragm) vibrates in response to sound wave.

Battery maintains a constant potential difference between the plates.

Processing circuit converts current into a varying output voltage.
17.6 Dielectrics.

There is a problem inherent in trying to store a large charge in a capacitor. To store a large charge without making the potential difference excessively large, we need a large capacitance.

Capacitance is inversely proportional to the spacing $d$ between the plates. One problem with making the spacing small is that the air between the plates of the capacitor breaks down at an electric field of about 3000 V/mm with dry air (less for humid air).

The breakdown allows a spark to jump across the gap so the stored charge is lost.
One way to overcome this difficulty is to put a better insulator than air between the plates.

Some insulating materials, which are also called dielectrics, can withstand electric fields larger than those that cause air to break down and act as a conductor rather than as an insulator.

Another advantage of placing a dielectric between the plates is that the capacitance itself is increased.

(And the dielectric keeps the plates from touching!)
Parallel Plate Capacitor with Dielectric

For a parallel plate capacitor in which a dielectric fills the entire space between the plates, the capacitance is.

**Capacitance of parallel plate capacitor with dielectric:**

\[ C = \kappa \frac{\varepsilon_0 A}{d} = \kappa \frac{A}{4\pi kd} \]

The effect of the dielectric is to increase the capacitance by a factor \( \kappa \) (Greek letter kappa), which is called the **dielectric constant**.
Dielectric Strength

The dielectric constant depends on the insulating material used.

The *dielectric strength* is the electric field strength at which *dielectric breakdown* occurs and the material becomes a conductor.

Since $\Delta V = Ed$ for a uniform field, the dielectric strength determines the maximum potential difference that can be applied across a capacitor per meter of plate spacing.
## Selected Dielectric Constants and Strengths

**Table 17.1** Dielectric Constants and Dielectric Strengths for Materials at 20°C (in Order of Increasing Dielectric Constant).

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength (kV/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1 (exact)</td>
<td>—</td>
</tr>
<tr>
<td>Air (dry, 1 atm)</td>
<td>1.00054</td>
<td>3</td>
</tr>
<tr>
<td>Paraffined paper</td>
<td>2.0–3.5</td>
<td>40–60</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Rubber (vulcanized)</td>
<td>3.0–4.0</td>
<td>16–50</td>
</tr>
<tr>
<td>Paper (bond)</td>
<td>3.0</td>
<td>8</td>
</tr>
<tr>
<td>Mica</td>
<td>4.5–8.0</td>
<td>150–220</td>
</tr>
</tbody>
</table>

Do not confuse dielectric constant and dielectric strength; they are not related.
Polarization in a Dielectric

What is happening microscopically to a dielectric between the plates of a capacitor?

Recall that polarization is a separation of the charge in an atom or molecule. The atom or molecule remains neutral, but the center of positive charge no longer coincides with the center of negative charge.
Polarization in a Dielectric

The charges on the capacitor plates induce a polarization of the dielectric.

The polarization occurs throughout the material, so the positive charge is slightly displaced relative to the negative charge.
Polarization in a Dielectric

Throughout the bulk of the dielectric, there are still equal amounts of positive and negative charge.

The net effect of the polarization of the dielectric is a layer of positive charge on one face and negative charge on the other. Each conducting plate faces a layer of opposing charge.
Dielectric Strength

The layer of opposing charge induced on the surface of the dielectric helps attract more charge to the conducting plate, for the same potential difference, than would be there without the dielectric.

Since capacitance is charge per unit potential difference, the capacitance must have increased.

The dielectric constant of a material is a measure of the ease with which the insulating material can be polarized.
Polarization in a Dielectric.

Start with a charge, $Q$, on the capacitor (and isolate the capacitor so $Q$ is fixed).

The induced charge on the faces of the dielectric create a polarization field, $E_{\text{POL}}$, that opposes the initial field, $E_0$.

The net electric field will be smaller: $E = E_0 - E_{\text{POL}}$. And so, for the amount of charge, $Q$, the voltage will be smaller and so the capacitance will be larger.

The dielectric constant, $K$, is the proportional increase in capacity.
The problem with this idea is that you almost NEVER have an isolated capacitor. It’s USUALLY in a circuit, at a fixed voltage, NOT a fixed charge.

!! Your mission, should you choose to accept it, is to explain how this model STILL works even if the VOLTAGE is constant instead of the charge. (HINT: The figure is still correct, but note that the voltage is the same with or without the dielectric. And so the net electric field is the same. For this to be true, what can you say about the charge on the capacitor?)
Suppose a dielectric is immersed in an external electric field $E_0$. The definition of the **dielectric constant** is the ratio of the electric field in vacuum $E_0$ to the electric field $E$ inside the dielectric material:

$$\kappa = \frac{E_0}{E}$$

The electric field inside the dielectric ($E$) is:

$$E = \frac{E_0}{\kappa}$$

$$\Delta V = \Delta V_0 / \kappa$$
Example 17.10

A parallel plate capacitor has plates of area 1.00 m² and spacing of 0.500 mm. The insulator has dielectric constant 4.9 and dielectric strength 18 kV/mm.

(a) What is the capacitance?

(b) What is the maximum charge that can be stored on this capacitor?
Example 17.10 Solution

(a) 

\[ C = \kappa \frac{A}{4\pi kd} \]

\[ = 4.9 \times \frac{1.00 \text{ m}^2}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times 5.00 \times 10^{-4} \text{ m}} \]

\[ = 8.67 \times 10^{-8} \text{ F} = 86.7 \text{ nF} \]

(b) 

\[ \Delta V = 18 \text{ kV/mm} \times 0.500 \text{ mm} = 9.0 \text{ kV} \]

\[ Q = C \Delta V = 8.67 \times 10^{-8} \text{ F} \times 9.0 \times 10^3 \text{ V} = 7.8 \times 10^{-4} \text{ C} \]
Example 17.11

A neuron can be modeled as a parallel plate capacitor, where the membrane serves as the dielectric and the oppositely charged ions are the charges on the “plates”.

Find the capacitance of a neuron and the number of ions (assumed to be singly charged) required to establish a potential difference of 85 mV. Assume that the membrane has a dielectric constant of $\kappa = 3.0$, a thickness of 10.0 nm, and an area of $1.0 \times 10^{-10}$ m$^2$. 
Example 17.11 Strategy

Since we know \( \kappa, A, \) and \( d, \) we can find the capacitance.

Then, from the potential difference and the capacitance, we can find the magnitude of charge \( Q \) on each side of the membrane.

A singly charged ion has a charge of magnitude \( e, \) so \( Q/e \) is the number of ions on each side.
Example 17.11 Solution

\[ C = \kappa \frac{A}{4\pi kd} \]

\[
C = 3.0 \times \frac{1.0 \times 10^{-10} \text{ m}^2}{4\pi \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \times 10.0 \times 10^{-9} \text{ m}} \\
= 2.66 \times 10^{-13} \text{ F} = 0.27 \text{ pF}
\]

\[
Q = C \Delta V = 2.66 \times 10^{-13} \text{ F} \times 0.085 \text{ V} \\
= 2.26 \times 10^{-14} \text{ C} = 0.023 \text{ pC}
\]

number of ions = \[
\frac{2.26 \times 10^{-14} \text{ C}}{1.602 \times 10^{-19} \text{ C} / \text{ion}} = 1.4 \times 10^5 \text{ ions}
\]
Application: Thunderclouds and Lightning

Access the text alternative for these images
17.7 Energy Stored in a Capacitor.

The energy stored in the capacitor can be found by summing the work done by the battery to separate the charge. (NOTE that the energy extracted from the battery is: $Q \Delta V$)
17.7 Energy Stored in a Capacitor

Suppose we look at this process at some instant of time when one plate has charge $+q_i$, the other has charge $-q_i$, and the potential difference between the plates is $\Delta V_i$.

If $\Delta q_i$ is small,

$$\Delta U_i = \Delta q_i \times \Delta V_i$$

$$U = \sum \Delta U_i = \sum \Delta q_i \times \Delta V_i$$
17.7 Energy Stored in a Capacitor

\[ U = \text{area of triangle} \times \frac{1}{2} (\text{base} \times \text{height}) \]

\[ U = \frac{1}{2} Q \Delta V \]

\[ \Delta V_t = \frac{q_t}{C} \]

\[ \Delta q, \, Q, \, q_i \]

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Example 17.12

Fibrillation is a chaotic pattern of heart activity that is ineffective at pumping blood and is therefore life-threatening.

A device known as a defibrillator is used to shock the heart back to a normal beat pattern. The defibrillator discharges a capacitor through paddles on the skin, so that some of the charge flows through the heart.

(a) If an 11.0-μF capacitor is charged to 6.00 kV and then discharged through paddles into a patient’s body, how much energy is stored in the capacitor?

(b) How much charge flows through the patient’s body if the capacitor discharges completely?
Example 17.12 Strategy

There are three equivalent expressions for energy stored in a capacitor.

Since the capacitor is completely discharged, all of the charge initially on the capacitor flows through the patient’s body.
Example 17.12 Solution

(a) \[ U = \frac{1}{2} Q \Delta V = \frac{1}{2} (C \Delta V) \times \Delta V = \frac{1}{2} C (\Delta V)^2 \]

\[ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \times 11.0 \times 10^{-6} \text{ F} \times (6.00 \times 10^3 \text{ V})^2 = 198 \text{ J} \]

(b) \[ Q = C \Delta V = 11.0 \times 10^{-6} \text{ F} \times 6.00 \times 10^3 \text{ V} = 0.0660 \text{ C} \]
Example: Charging while idling

How much energy in your gasoline is wasted by charging up your car?

The energy stored by charging up your car must come from somewhere, and the only source of energy in your car is the gasoline. The energy stored in the charging of your car is:

\[ \Delta U = \frac{1}{2} Q \Delta V = (0.0012 \, C)(6 \cdot 10^6 \, V) = 7200 \, J \]

Seems like a lot, but a gallon of gas has an energy content of about 127 MJ (https://en.wikipedia.org/wiki/Gasoline)

So this probably too-large amount is only 0.000004 % of the energy in your tank...
Energy Stored in an Electric Field.

Potential energy is energy of interaction or field energy.

The energy stored in a capacitor is stored in the electric field between the plates. We can use the energy stored in a capacitor to calculate how much energy per unit volume is stored in an electric field $E$.

Why energy per unit volume? Two capacitors can have the same electric field but store different amounts of energy. The larger capacitor stores more energy, proportional to the volume of space between the plates.
Energy Stored in an Electric Field.

Another way to think of it is that I can think of the energy as work done while moving charges around (as we’ve discussed) OR as the energy stored when I separate the charges and create an electric field.

Which is choose is, at the moment, a matter of convenience (and the answer has to be the same either way)

BUT when we get to magnetism, energy is ONLY conserved if SOME of the energy is in the electric and magnetic field. Then we HAVE to worry about this.
Energy Stored in an Electric Field

In a parallel plate capacitor, the energy stored is.

\[ U = \frac{1}{2} C (\Delta V)^2 + \frac{1}{2} \kappa \frac{A}{4\pi kd} (\Delta V)^2 \]

Assuming the field is uniform between the plates, the potential difference is.

\[ \Delta V = Ed \]

\[ U = \frac{1}{2} \kappa \frac{A}{4\pi kd} (E_d)^2 = \frac{1}{2} \kappa \frac{Ad}{4\pi k} E^2 \]

Then the energy density \( u \)—the electric potential energy per unit volume—is.

\[ u = \frac{U}{Ad} = \frac{1}{2} \kappa \frac{1}{4\pi k} E^2 = \frac{1}{2} \kappa \epsilon_0 E^2 \]
EXAMPLE: Energy from the Sun

If I divide the “brightness” of sunlight (power/area) by the speed of light (m/s), we get the energy density (J/m³)

!! Show using the units that this is true.

\[ u = \frac{1360 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 4.5 \cdot 10^{-6} \text{ J/m}^3 \]

Using the expression for the energy density:

\[ E = \sqrt{\frac{2u}{\varepsilon_0}} = \sqrt{\frac{9 \cdot 10^{-6} \text{ J/m}^3}{8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)}} = 10^3 \frac{N}{C} = 10^3 \frac{V}{m} \]

!! Show that the units under the radical give both N/C and V/m