Chapter 16: Electric Forces and Fields

16.1 Electric Charge.
16.2 Electric Conductors and Insulators.
16.3 Coulomb’s “law”.
16.4 The Electric Field.
16.5 Motion of a Point Charge in a Uniform Electric Field.
16.6 Conductors in Electrostatic Equilibrium.
16.7 Gauss’s “law” for Electric Fields.
Until the invention of the battery (Alessandro Volta) electricity experiments were basically static electricity, like when your clothes cling out of the dryer or you shock yourself after walking across a carpet.

One early worker was Benjamin Franklin (1750s). In his book on electricity, he describes:

- killing turkeys
- Proposes execution
- Knocked down up to 6 men holding hands (by essentially building up charge by rubbing)
A fluid model of electricity proposed that when you rub, you move charge FROM one object TO another, so the NET amount was fixed. (Franklin, et al., proposed ONLY one fluid).

So as you rubbed objects together, there was a DEFICIT of charge on one and an EXCESS on the other:
“Whenever a physicists makes a 50/50 guess, they’ll be wrong about 90% of the time” Dr Wm Deering, UNT

MUCH later experiments showed a need for TWO kinds of charge

J J Thomson (1897) showed there was a negatively charged particle called the electron.
Elementary Charge

The magnitude of charge on the proton and electron is the same. That amount of charge is called the elementary charge (symbol $e$).

$$e = 1.602 \times 10^{-19} \text{ C}$$

The net charge of any object is an integral multiple of the elementary charge.
# Proton, Electron, and Neutron

Table 16.1 Masses and Electric Charges of the Proton, Electron, and Neutron.

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<td>$m_n = 1.675 \times 10^{-27}$ kg</td>
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Note that the mass of the electron is MUCH less – that is one reason the electrons usually are the charge that moves around – it’s just easier to move. (Another is that electrons are bound by the electric force and protons are bound by the MUCH STRONGER nuclear force).
# Proton, Electron, and Neutron

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Protons and neutrons are NOT fundamental, but are made of three quarks, each charged at $\pm e/3$ or $\pm 2e/3$. So the proton is could be made up of three $+e/3$ quarks – so the charge is $+e/3 + e/3 + e/3 = +e$:

$->$ Are there other ways to arrange three quarks to get a total charge of $+e$?

How about the neutron? How many ways are there to combine three quarks to get a net charge of zero?

(Clearly you can google this – and if that’s all you ever do, you’ll be well prepared for a career as a “googler” – I wonder how many of those there are?)
This was discovered by firing a beam of electrons at a proton and looking at how they bounced off of the proton.
Example 16.1

The magnitude of charge transferred when you walk across a carpet, reach out to shake hands, and unintentionally give a shock to a friend might be typically about 1 nC.

(a) If the charge is transferred by electrons only, how many electrons are transferred?

(b) If your body has a net charge of $-1$ nC, estimate the percentage of excess electrons.

[Hint: The mass of the electron is only about 1/2000 that of a nucleon, so most of the mass of the body is in the nucleons. For an order-of-magnitude calculation, we can just assume that half of the nucleons are protons and half are neutrons. Look at the periodic table...].
Example 16.1 Strategy

Since the coulomb (C) is the SI unit of charge, the “n” must be the prefix “nano-” \((= 10^{-9})\). We know the value of the elementary charge in coulombs.

For part (b), we first make an order-of-magnitude estimate of the number of electrons in the human body.
Example 16.1 Solution

\[
\frac{-1 \times 10^{-9} \text{C}}{-1.6 \times 10^{-19} \text{C per electron}} = 6 \times 10^9 \text{ electrons}
\]

number of nucleons = \(\frac{\text{mass of body}}{\text{mass per nucleon}} = \frac{70 \text{ kg}}{1.7 \times 10^{-27} \text{ kg}}\)

= \(4 \times 10^{28}\) nucleons

number of protons = \(\frac{1}{2} \times 4 \times 10^{28} = 2 \times 10^{28}\) protons

The percentage of excess electrons is then.

\[
\frac{6 \times 10^9}{2 \times 10^{28}} \times 100\% = \left(3 \times 10^{-17}\right)\%
\]
Example – charging by idling your engine

As your engine runs, it exhausts gasses out the tailpipe. Many are ions (SO$_4^{2-}$ or NO$_3^-$). Since charge is conserved, the car (and its contents) must then be positively charged. This is why you sometimes get shocked as you get out of a car and touch something (or VERY RARELY, ignite the gas when you fill up a car at a gas station). We can roughly estimate how fast a car charges while idling:

A full tank empties in about 30 hours of idling. And gas has a density of about 0.75 kg/L. So gas burns in an engine at a rate of:

\[
15 \text{gal}/30 \text{ hours} \times 1 \text{ hr}/60 \text{ min} \times 4 \text{L}/\text{gal} \times 0.75 \text{ kg/L}
\]
Example – charging by idling your engine

$$15\text{gal} / 30\text{ hours} \times 1\text{hr} / 60\text{ min} \times 4\text{L} / \text{gal} \times 0.75\text{ kg/L}$$

$$= 25\text{ g/min}$$

EPA requires less than 10 ppm of sulfur in gasoline, so less than 250 $\mu$g/min of sulfur leaving the tailpipe.

Using 32 g/mol of sulfur:

$$250\,\mu\text{g/min} \times 1\text{ mole} / 32\text{ g} \times 6\times10^{23}/\text{mole} \times 2 \times 1.6\times10^{-19}\text{ C}$$

$$= 1.5\text{ C / minute}$$

This implies that in a 10 minute drive, the car charges to about 15 C – we’ll find that this is a large overestimate.

(But gas pump fires are NOT caused by young people with phones…. )
Attraction, Repulsion, and Polarization

Like charges repel one another; unlike charges attract one another.

Polarization.

An electrically neutral object may have regions of positive and negative charge within it, separated from one another. Such an object is polarized.

A polarized object can experience an electric force even though its net charge is zero.
The net force on the paper is always attractive, regardless of the sign of charge on the rod. In this case, we say that the paper is polarized by induction.
Polarized Molecules

Some molecules (e.g., water) are intrinsically polarized.
Application: Hydrogen Bonds in Water
Due largely to hydrogen bonding, water:

- is a liquid rather than a gas at room temperature;
- has a large specific heat;
- has a large heat of vaporization;
- is less dense as a solid (ice) than as a liquid;
- has a large surface tension;
- exhibits strong adhesive forces with some surfaces;
- is a powerful solvent of polar molecules.
Application: Hydrogen Bonds in DNA, RNA, and Proteins

Hydrogen bonds

Adenine

Thymine
16.2 Electric Conductors and Insulators

The difference is in HOW the atoms bond (chemistry) because ALL are the same inside – almost exactly as many electrons as protons. Difference is whether a current (collective motion of charge) can be generated.

Materials which support a current are called electric conductors, whereas materials which don’t are called electrical insulators.

Intermediate between conductors and insulators are the semiconductors.
Charging Insulators by Rubbing

When different insulating objects are rubbed against one another, both electrons and ions (charged atoms) can be transferred from one object to the other. (Difference in electron affinity – chemistry – makes charge move from one to the other).
Charging a Conductor by Contact

Touch a charged insulator to the conductor. Since the charge transferred to the conductor spreads out, the process can be repeated to build up more and more charge on the conductor.
Grounding a Conductor

How can a conductor be discharged? One way is to ground it.

Earth is a conductor because of the presence of ions and moisture, and it’s large enough that for many purposes it can be thought of as a limitless reservoir of charge.

To ground a conductor means to provide a conducting path between it and the Earth (or to another charge reservoir). A charged conductor that is grounded discharges because the charge spreads out by moving off the conductor and onto the Earth.
Charging a Conductor by Induction

A conductor is not necessarily discharged when it is grounded if there are other charges nearby.

It is even possible to charge an initially neutral conductor by grounding it.
Charging a Conductor by Induction

(a) Glass rod
(b) Metal sphere
(c) Electrons flowing from ground through wire to sphere
(d) Disconnecting ground wire
(e) Equilibrium attained
Example 16.2

An electroscope is charged negatively and the gold foil leaves hang apart.

What happens to the leaves as the following operations are carried out in the order listed? Explain what you see after each step.

(a) You touch the metal bulb at the top of the electroscope with your hand.

(b) You bring a glass rod that has been rubbed with silk near the bulb without touching it. [Hint: A glass rod rubbed with silk is positively charged.].

(c) The glass rod touches the metal bulb.
Example 16.2 Solution 1

(a) By touching the electroscope bulb with your hand, you ground it. Charge is transferred between your hand and the bulb until the bulb’s net charge is zero. Since the electroscope is now discharged, the foil leaves hang down.
Example 16.2 Solution 2

(b) When the positively charged rod is held near the bulb, the electroscope becomes polarized by induction. Negatively charged free electrons are drawn toward the bulb, leaving the foil leaves with a positive net charge. The leaves hang apart due to the mutual repulsion of the net positive charges on them.
Example 16.2 Solution 3

(c) When the positively charged rod touches the bulb, some negative charge is transferred from the bulb to the rod.

The electroscope now has a positive net charge.

The glass rod still has a positive net charge that repels the positive charge on the electroscope, pushing it as far away as possible—toward the foil leaves.

The leaves hang farther apart, since they now have more positive charge on them than before.
Application: Photocopiers and Laser Printers

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Coulomb’s “law” gives the electric force acting between two point charges.

A point charge is a point-like object with a nonzero electric charge.

Like gravity, the electric force is an inverse square law force.

Even for charges that are NOT points, Coulomb’s “law” gives a rough number for the force.
Magnitude of Electric Force

The magnitude of the electric force that each of two charges exerts on the other is given by:

\[ F = \frac{k|q_1||q_2|}{r^2} \]

The constant \( k \), which we call the Coulomb constant, can be written in terms of another constant \( \varepsilon_0 \), the permittivity of free space:

\[ k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \text{C}^2 \]

\[ \varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ N} \cdot \text{m}^2 \text{C}^2 \]
Direction of Electric Force

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(a) 
\[ \vec{F}_{12} \quad q_1 \quad \vec{F}_{21} \quad q_2 \]

(b) 
\[ \vec{F}_{12} \quad q_1 \quad \vec{F}_{21} \quad q_2 \]

(c) 
\[ \vec{F}_{12} \quad q_1 \quad \vec{F}_{21} \quad q_2 \]
1. Use consistent units; since we know $k$ in standard SI units ($N \cdot m^2/C^2$), distances should be in meters and charges in coulombs. When the charge is given in $\mu C$ or $nC$, be sure to change the units to coulombs:

$$1 \, \mu C = 10^{-6} \, C \text{ and } 1 \, nC = 10^{-9} \, C.$$
2. When finding the electric force on a single charge due to two or more other charges, find the force due to each of the other charges separately.

The net force on a particular charge is the vector sum of the forces acting on that charge due to each of the other charges.

Often it helps to separate the forces into $x$- and $y$-components, add the components separately, then find the magnitude and direction of the net force from its $x$- and $y$-components.
3. If several charges lie along the same line, do not worry about an intermediate charge “shielding” the charge located on one side from the charge on the other side.

The electric force is long-range just as is gravity; the gravitational force on the Earth due to the Sun does not stop when the Moon passes between the two.
Example – force on two charged people.

We’ll find that a typical value for the charge you get (from walking across a carpet or sitting in a running car) is about 1 nC. What is the magnitude of the electric force they exert?

\[ F = \left( 9 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( 10^{-9} \text{ C} \right) \left( 10^{-9} \text{ C} \right) / (3 \text{ m})^2 \approx 10^{-9} \text{ N} \]

Compare this to the gravitational force they exert?

\[ F = \left( 7 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \left( 70 \text{ kg} \right) \left( 70 \text{ kg} \right) / (3 \text{ m})^2 \approx 4 \cdot 10^{-8} \text{ N} \]
Example 16.3

Suppose three point charges are arranged as follows. A charge \( q_1 = +1.2 \ \mu\text{C} \) is located at the origin of an \((x, y)\) coordinate system; a second charge \( q_2 = -0.60 \ \mu\text{C} \) is located at \((1.20 \ \text{m}, 0.50 \ \text{m})\) and the third charge \( q_3 = +0.20 \ \mu\text{C} \) is located at \((1.20 \ \text{m}, 0)\). What is the force on \( q_2 \) due to the other two charges?
Example 16.3 Strategy

The force on $q_2$ due to $q_1$ and the force on $q_2$ due to $q_3$ are determined separately.

After sketching a free-body diagram, we add the two forces as vectors.

Let the distance between charges 1 and 2 be $r_{12}$ and the distance between charges 2 and 3 be $r_{23}$. 
Example 16.3 Solution 1

\[ q_1 \quad \rightarrow \quad q_2 \quad \rightarrow \quad q_3 \]

\[ \vec{F}_{21} \quad \theta \quad \vec{F}_{23} \]

\[ \vec{F}_{21} \quad \vec{F}_{23} \quad \vec{F}_2 \]
Example 16.3 Solution 2

\[ r_{12} = \sqrt{r_{13}^2 + r_{23}^2} = 1.30 \text{ m} \]

\[ F_{21} = \frac{k|q_1|q_2}{r_{12}^2} \]

\[ = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \frac{(1.2 \times 10^{-6} \text{ C}) \times (0.60 \times 10^{-6} \text{ C})}{(1.30 \text{ m})^2} \]

\[ = 3.83 \times 10^{-3} \text{ N} = 3.83 \text{ mN} \]
Example 16.3 Solution 3

\[ F_{23} = \frac{k |q_2||q_3|}{r_{23}^2} \]

\[ = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \frac{(0.20 \times 10^{-6} \text{C}) \times (0.60 \times 10^{-6} \text{C})}{(0.50 \text{ m})^2} \]

\[ = 4.32 \times 10^{-3} \text{ N} = 4.32 \text{ mN} \]
Example 16.3 Solution 4

\[ F_{21x} = -F_{21} \sin \theta = -3.83 \text{ mN} \times \frac{1.20 \text{ m}}{1.30 \text{ m}} = -3.53 \text{ mN} \]

\[ F_{21y} = -F_{21} \cos \theta = -3.83 \text{ mN} \times \frac{0.50 \text{ m}}{1.30 \text{ m}} = -1.47 \text{ mN} \]

\[ F_{23x} = 0 \text{ and } F_{23y} = -4.32 \text{ mN} \]

\[ F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = 6.8 \text{ mN} \]

\[ \phi = \tan^{-1} \left( \frac{3.53 \text{ mN}}{5.79 \text{ mN}} \right) = 31^\circ \]
Example 16.4

Two Styrofoam balls of mass 10.0 g are suspended by threads of length 25 cm. The balls are charged, after which they hang apart, each at $\theta = 15.0^\circ$ to the vertical.

(a) Are the signs of the charges the same or opposite?

(b) Are the magnitudes of the charges necessarily the same? Explain.

(c) Find the net charge on each ball, assuming that the charges are equal.
Example 16.4 Strategy

The situation is similar to the charged electroscope. Each ball exerts an electric force on the other since both are charged.

The gravitational forces that the balls exert on one another are negligibly small, but the gravitational forces that Earth exerts on the balls are not negligible.

The third force acting on each of the balls is due to the tension in a thread. We analyze the forces acting on a ball using an FBD. The sum of the three forces must be zero since the ball is in equilibrium.
Example 16.4 Solution 1

(a) The electric force is clearly repulsive—the balls are pushed apart—so the charges must have the same sign. There is no way to tell whether they are both positive or both negative.

(b) The force on either of the balls is proportional to the product of the two charge magnitudes; \( F \propto q_1 q_2 \). In accordance with Newton’s third “law”, Coulomb’s “law” says that the two forces that make up the interaction are equal in magnitude and opposite in direction. The charges are not necessarily equal.
Example 16.4 Solution 2

(c) \[
\begin{align*}
\sum F_x &= F_E - T \sin \theta = 0 \\
\sum F_y &= T \cos \theta - mg = 0
\end{align*}
\]

\[F_E = T \sin \theta = \left( \frac{mg}{\cos \theta} \right) \sin \theta = mg \tan \theta\]

\[F_E = \frac{k |q|^2}{r^2} \quad r = 2(d \sin \theta) \quad d = 25 \text{ cm}\]

\[|q|^2 = \frac{F_E r^2}{k}\]
Example 16.4 Solution 3

(c) continued.

\[ |q|^2 = \frac{F_E r^2}{k} \]

\[ |q|^2 = \frac{(mg \tan \theta)(2d \sin \theta)^2}{k} \]

\[ = \frac{4d^2 mg \tan \theta \sin^2 \theta}{k} \]

\[ |q| = \sqrt{\frac{4 \times (0.25 \text{ m})^2 \times 0.0100 \text{ kg} \times 9.8 \text{ N/kg} \times \tan 15.0^\circ \times \sin^2 15.0^\circ}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} \]

\[ = 0.22 \mu \text{C} \]

The charges can either both be positive or both negative, so the charges are either both +0.22 µC or both -0.22 µC.
16.4 The Electric Field

If a point charge $q$ is in the vicinity of other charges, it experiences an electric force, $\vec{F}_E$.

The **electric field at any point** is defined to be the electric force **per unit charge** at that point. The electric field is denoted $\vec{E}$.
16.4 The Electric Field

The SI units of the electric field are N/C.

(Much like the gravitational field: which can be used in oil prospecting)

Why Define an Electric Field?

One reason is that once we know the electric field at some point, then it is easy to calculate the electric force on any point charge $q$ placed there:
16.4 The Electric Field

Why Define an Electric Field?

Another reason is that to calculate the FORCE we need both charges ($q_1$ and $q_2$); to calculate the field, we will see we only need one – useful when we design:
antennas; cell phones; WiFi routers; radar; etc
Example 16.5

A small sphere of mass 5.10 g is hanging vertically from an insulating thread that is 12.0 cm long. By charging some nearby flat metal plates, the sphere is subjected to a horizontal electric field of magnitude $7.20 \times 10^5$ N/C. As a result, the sphere is displaced 6.00 cm horizontally in the direction of the electric field.

(a) What is the angle $\theta$ that the thread makes with the vertical?

(b) What is the tension in the thread?

(c) What is the charge on the sphere?
Example 16.5 Strategy

We assume that the sphere is small enough to be treated as a point charge.

Then the electric force on the sphere is given by $\mathbf{F}_E = q\mathbf{E}$.

The figure shows that the sphere is pushed to the right by the field; therefore, the electric force is to the right. Since the electric field and force have the same direction, the charge on the sphere is positive.

After drawing an FBD showing all the forces acting on the sphere, we set the net force on the sphere equal to zero since it hangs in equilibrium.
Example 16.5 Solution 1

(a)

\[ \sin \theta = \frac{6.00 \text{ cm}}{12.0 \text{ cm}} = 0.500 \]

and

\[ \theta = 30.0^\circ \]
Example 16.5 Solution 2

(b)

\[ \sum F_y = T \cos \theta - mg = 0 \]

\[ T = \frac{mg}{\cos \theta} = \frac{5.10 \times 10^{-3} \text{ kg} \times 9.80 \text{ N/kg}}{\cos 30.0^\circ} = 0.0577 \text{ N} \]
Example 16.5 Solution 3

(c) \[
\sum F_x = |q| E - T \sin \theta = 0
\]

\[
|q| = \frac{T \sin \theta}{E} = \frac{(5.77 \times 10^{-2} \text{ N}) \sin 30.0^\circ}{7.20 \times 10^5 \text{ N/C}} = 40.1 \text{ nC}
\]

\[
q = 40.1 \text{ nC}
\]
Electric Field due to a Point Charge

The electric field due to a single point charge $Q$ can be found using Coulomb’s “law”.

Imagine a positive test charge $q$ placed at various locations. Coulomb’s “law” says that the force acting on the test charge is.

\[
F = \frac{k|q||Q|}{r^2}
\]

The electric field strength is then.

\[
E = \frac{F}{|q|} = \frac{k|Q|}{r^2}
\]
Principle of Superposition

The electric field at any point is the vector sum of the field vectors at that point caused by each charge separately.
Example 16.6

Two point charges are located on the $x$-axis. Charge $q_1 = +0.60 \, \mu\text{C}$ is located at $x = 0$; charge $q_2 = -0.50 \, \mu\text{C}$ is located at $x = 0.40 \, \text{m}$. Point $P$ is located at $x = 1.20 \, \text{m}$.

What is the magnitude and direction of the electric field at point $P$ due to the two charges?
Example 16.6 Strategy

We can determine the field at $P$ due to $q_1$ and the field at $P$ due to $q_2$ separately using Coulomb’s “law” and the definition of the electric field.

In each case, the electric field points in the direction of the electric force on a positive test charge at point $P$.

The sum of these two fields is the electric field at $P$. 
Example 16.6 Solution 1

\[ E_1 = \frac{k |q_1|}{r_1^2} \]

\[ = 8.99 \times 10^9 \, \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \frac{0.60 \times 10^{-6} \, \text{C}}{(1.20 \, \text{m})^2} \]

\[ = 3.75 \times 10^3 \, \text{N/C} \]
Example 16.6 Solution 2

\[ \vec{E}_2 = \frac{k |q_2|}{r_2^2} \]

\[ = \frac{8.99 \times 10^9 \ \text{N} \cdot \text{m}^2}{\text{C}^2} \times \frac{0.50 \times 10^{-6} \ \text{C}}{(0.80 \ \text{m})^2} \]

\[ = 7.02 \times 10^3 \ \text{N/C} \]
Example 16.6 Solution 3

The electric field at $P$ is $3.3 \times 10^3$ N/C in the $-x$-direction.
Example 16.7

Three point charges are placed at the corners of a rectangle, as shown in the figure.

(a) What is the electric field due to these three charges at the fourth corner, point \( P \)?

(b) What is the acceleration of an electron located at point \( P \)? Assume that no forces other than that due to the electric field act on it.
Example 16.7 Strategy

(a) After determining the magnitude and direction of the electric field at point $P$ due to each point charge individually, we use the principle of superposition to add them as vectors.

(b) Since we have already calculated the electric field at point $P$, the force on the electron is given by $\mathbf{F} = q \mathbf{E}$, where $q = -e$ is the charge of the electron.
Example 16.7 Solution 1

(a)

\[
E_1 = \frac{k |q_1|}{r_1^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \times 4.0 \times 10^{-6} \text{ C}}{(0.50 \text{ m})^2} = 1.44 \times 10^5 \times \text{N/C}
\]

\[
E_1 = \frac{k |q_1|}{r_1^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \times 4.0 \times 10^{-6} \text{ C}}{(0.50 \text{ m})^2} = 1.44 \times 10^5 \times \text{N/C}
\]

A similar calculation with \( |q_3| = 1.0 \times 10^{-6} \text{ C} \) and \( r_3 = 0.20 \text{ m} \) yields \( E_3 = 2.25 \times 10^5 \text{ N/C} \). Using the Pythagorean theorem to find \( r_2 = \sqrt{(0.50 \text{ m})^2 + (0.20 \text{ m})^2} \), we have

\[
E_2 = \frac{k q_2}{r_2^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \times 6.0 \times 10^{-6} \text{ C}}{(0.50 \text{ m})^2 + (0.20 \text{ m})^2} = 1.86 \times 10^5 \text{ N/C}
\]
Example 16.7 Solution 2

(a) continued.

\[
\begin{align*}
\cos \theta &= \frac{r_1}{r_2} = 0.928 \\
\sin \theta &= 0.371 \\
\end{align*}
\]

\[
\sum E_x = E_{1x} + E_{2x} + E_{3x} = (-E_1) + (-E_2 \cos \theta) + 0 = -3.17 \times 10^5 \text{ N/C}
\]

\[
\sum E_y = E_{1y} + E_{2y} + E_{3y} = 0 + E_2 \sin \theta - E_3 = -1.56 \times 10^5 \text{ N/C}
\]

The magnitude of the electric field is then \(E = \sqrt{E_x^2 + E_y^2} = 3.5 \times 10^5 \text{ N/C}\) and the direction is at angle \(\phi = \tan^{-1} \left| \frac{E_y}{E_x} \right| = 26^\circ\) below the \(-x-\)axis.
Example 16.7 Solution 3

(b) The force on the electron is $F = qE$. Its acceleration is then $a = \frac{q_e E}{m_e}$. The electron charge $q_e = -e$ and mass $m_e$ are given in Table 16.1. The acceleration has magnitude $a = eE/m_e = 6.2 \times 10^{16}$ m/s$^2$. The direction of the acceleration is the direction of the electric force, which is opposite the direction of $E$ since the electron’s charge is negative.

Notice how CRAZY big the acceleration is – that’s not a misprint – electron accelerations are HUGE under very reasonable circumstances.
Electric Field Lines

It is often difficult to make a visual representation of an electric field using vector arrows; the vectors drawn at different points may overlap and become impossible to distinguish.

Another visual representation of the electric field is a sketch of the **electric field lines**, a set of continuous lines that represent both the magnitude and the direction of the electric field vector as follows. (Invented by Michael Faraday, who was a “visual learner”)

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Interpretation of Electric Field Lines

The direction of the electric field vector at any point is *tangent to the field line* passing through that point and in the direction indicated by arrows on the field line.

The electric field is strong where field lines are close together and weak where they are far apart.
Rules for Sketching Field Lines

Electric field lines can start only on positive charges and can end only on negative charges.

The number of lines starting on a positive charge (or ending on a negative charge) is proportional to the magnitude of the charge. (The total number of lines you draw is arbitrary; the more lines you draw, the better the representation of the field.).
Rules for Sketching Field Lines

Field lines never cross. The electric field at any point has a unique direction; if field lines crossed, the field would have two directions at the same point.
Field Lines for a Point Charge

The figure shows sketches of the field lines due to single point charges. The field lines show that the direction of the field is radial (away from a positive charge or toward a negative charge).
Field Lines for a Point Charge

The lines are close together near the point charge, where the field is strong, and are more spread out farther from the point charge, showing that the field strength diminishes with distance.
Electric Field due to a Dipole

A pair of point charges with equal and opposite charges that are near one another is called a dipole (literally two poles).

To find the electric field due to the dipole at various points by using Coulomb’s “law” would be extremely tedious, but sketching some field lines immediately gives an approximate idea of the electric field (next slide).
Electric Field due to a Dipole

Electric Field due to a Dipole

Electric Field due to a Dipole

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Example 16.8

A thin metallic spherical shell of radius $R$ carries a total charge $Q$, which is positive. The charge is spread out evenly over the shell’s outside surface. Sketch the electric field lines in two different views of the situation:

(a) the spherical shell is tiny, and you are looking at it from distant points;

(b) you are looking at the field inside the shell’s cavity.

In (a), also sketch $\mathbf{E}$ field vectors at two different points outside the shell.
Example 16.8 Strategy

Since the charge on the shell is positive, field lines begin on the shell.

A sphere is a highly symmetrical shape: standing at the center, it looks the same in any chosen direction. This symmetry helps in sketching the field lines.
Example 16.8 Solution 1

(a)

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We can see later that NO field in a spherical shell means the electric field is PRECISELY inverse square.
Example 16.8 Solution 2

(b)

We can see later that NO field in a spherical shell means the electric field is PRECISELY inverse square.

\[ \vec{E} = \frac{kq}{r^2} \]

Is this EXACTLY 2 or just really close to 2?
Application of Electric Fields: Electrolocation
16.5 Motion of a Point Charge in a Uniform Electric Field

The simplest example of how a charged object responds to an electric field is when the electric field (due to other charges) is **uniform**—that is, has the same magnitude and direction at every point.

\[ E = \frac{Q}{\varepsilon_0 A} \]

\[ \vec{F} = q \vec{E} \]

\[ \vec{a} = \frac{\vec{F}}{m} = \frac{q \vec{E}}{m} \]
First, how to make a Uniform Electric Field

Imagine a large, flat plate carrying a positive charge $Q$, and consider the electric field at a point near the plate but NOT near any of the edges.

The NET field is AWAY from the plate and primarily due to charges that are “below” the point.
First, how to make a Uniform Electric Field

Farther from the plate, the field is weaker (inverse square) BUT MORE charge is “below” the point.

Since the electric force is inverse square \((1/r^2)\), these effects EXACTLY cancel. The field is UNIFORM.

!! What if the plate had a negative charge?
First, how to make a Uniform Electric Field

From units alone, the magnitude of the field must have things of the form:

$$E = \{\text{no units}\} k \ [\text{N m}^2/\text{C}^2] \ \text{charge [Coulombs]} / \ \text{distance squared [m}^2]$$

Distance squared – Can’t be a distance (would change E)

Use AREA of plate.
First, how to make a Uniform Electric Field

So this:

\[ E = \{\text{no units}\} k \ [N \ m^2/C^2] \ \text{charge} \ [\text{Coulombs}] \ / \ \text{distance squared} \ [m^2] \]

Becomes:

\[ E = \{2\pi\} k \ Q / A \]

\[ \vec{E} \]
First, how to make a Uniform Electric Field

Field outside is zero.
Field between is double.

\[ \vec{E} = \frac{4\pi k Q}{A} \]

\[ \vec{E} = 0 \]
Acceleration of a Charged Particle due to an Electric Field

\[ \vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} \]

Another example of constant force is a weight near the Earth’s surface (like you did in mechanics!) so the behavior is the same.

Except one thing: The direction of the acceleration is either parallel (for a positive charge) or antiparallel (for a negative charge) to the electric field. (No such thing as negative mass...as far as we know...)
Example 16.9 (1)

A cathode ray tube (CRT) is used to accelerate electrons in some televisions, computer monitors, oscilloscopes, and x-ray tubes. Electrons from a heated filament pass through a hole in the cathode; they are then accelerated by an electric field between the cathode and the anode (next slide).

Suppose an electron passes through the hole in the cathode at a velocity of $1.0 \times 10^5 \text{ m/s}$ toward the anode. The electric field is uniform between the anode and cathode and has a magnitude of $1.0 \times 10^4 \text{ N/C}$.

(a) What is the acceleration of the electron?

(b) If the anode and cathode are separated by 2.0 cm, what is the final velocity of the electron?
Example 16.9 (2)

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Example 16.9 Strategy

Given: initial speed $v_i = 1.0 \times 10^5$ m/s;
separation between plates $d = 0.020$ m;
electric field magnitude $E = 1.0 \times 10^4$ N/C.

Look up: electron mass $m_e = 9.109 \times 10^{-31}$ kg;
electron charge $q = -e = -1.602 \times 10^{-19}$ C.

Find: (a) acceleration; (b) final velocity.
Example 16.9 Solution 1

(a) \[ F_g = mg = 9.109 \times 10^{-31} \text{ kg} \times 9.8 \text{ m/s}^2 = 8.9 \times 10^{-30} \text{ N} \]

\[ F_E = eE = 1.602 \times 10^{-19} \text{ C} \times 1.0 \times 10^4 \text{ N/C} = 1.6 \times 10^{-15} \text{ N} \]

\[ a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{1.602 \times 10^{-19} \text{ C} \times 1.0 \times 10^4 \text{ N/C}}{9.109 \times 10^{-31} \text{ kg}} \]

\[ = 1.76 \times 10^{15} \text{ m/s}^2 \]

To two significant figures, \( a = 1.8 \times 10^{15} \text{ m/s}^2 \). Since the charge on the electron is negative, the direction of the acceleration is opposite to the electric field, or to the right in the figure.
Example 16.9 Solution 2

(b)

\[
v_f = \sqrt{v_i^2 + 2ad}
\]

\[
= \sqrt{(1.0 \times 10^5 \text{ m/s})^2 + 2 \times 1.76 \times 10^{15} \text{ m/s}^2 \times 0.020 \text{ m}}
\]

\[
= 8.4 \times 10^6 \text{ m/s to the right}
\]
Example 16.10

An electron is projected horizontally into the uniform electric field directed vertically downward between two parallel plates. The plates are 2.00 cm apart and are of length 4.00 cm. The initial speed of the electron is \( v_i = 8.00 \times 10^6 \) m/s. As it enters the region between the plates, the electron is midway between the two plates; as it leaves, the electron just misses the upper plate.

What is the magnitude of the electric field?
Example 16.10 Strategy

Given: $\Delta x = 4.00 \text{ cm}; \Delta y = 1.00 \text{ cm}; v_x = 8.00 \times 10^6 \text{ m/s}$

Find: electric field strength, $E$. 
Example 16.10 Solution

\[ \Delta t = \frac{\Delta x}{v_x} = \frac{4.00 \times 10^{-2} \text{ m}}{8.00 \times 10^{-1} \text{ m/s}} = 5.00 \times 10^{-9} \text{ s} \]

\[ \Delta y = \frac{1}{2} a_y (\Delta t)^2 \]

\[ a_y = \frac{2 \Delta y}{(\Delta t)^2} = \frac{2 \times 1.00 \times 10^{-2} \text{ m}}{(5.00 \times 10^{-9} \text{ s})^2} = 8.00 \times 10^{14} \text{ m/s}^2 \]

\[ F_y = qE_y = m_e a_y \]

\[ E_y = \frac{m_e a_y}{q} = \frac{9.109 \times 10^{-31} \text{ kg} \times 8.00 \times 10^{14} \text{ m/s}^2}{6.02 \times 10^{-19} \text{ C}} = -4.55 \times 10^3 \text{ N/C} \]

Since the field has no \( x \)-component, its magnitude is \( 4.55 \times 10^3 \text{ N/C} \).
EXAMPLE: Photoelectric effect

Heinrich Hertz discovered that electrons fly off a metal plate faster or slower according to the color of light you shine on the metal plate. How did he measure the speed?

\[ \vec{E} = 4 \pi \kappa \frac{Q}{A} \]

Detect electrons leaving through the hole and adjust the field between the plates until they no longer leave (final speed is zero).
EXAMPLE: Photoelectric effect

\[ v^2 = v_0^2 + 2a(x - x_0) \]

Use final \( v = 0 \)

\[ v_0 = \sqrt{-2a(x - x_0)} = \sqrt{-2ad} \]

Use Newton’s Second equation:

\[ a = \frac{F}{m} = \frac{q}{m} E = \frac{q}{m} \frac{Q}{\varepsilon_0 A} \]

\[ v_0 = \sqrt{-2 \left( \frac{q}{m} \right) \left( \frac{Q}{\varepsilon_0 A} \right) d} \]
The most easily polarized materials are conductors because they contain highly mobile charges that can move freely through the entire volume of the material.

It is useful to examine the distribution of charge in a conductor, whether the conductor has a net charge or lies in an externally applied field, or both.

We restrict our attention to a conductor in which the mobile charges are at rest in equilibrium, a situation called electrostatic equilibrium.
If charge is put on a conductor, mobile charges move about until a stable distribution is attained. The same thing happens when an external field is applied or changed—charges move in response to the external field, but they eventually reach an equilibrium distribution.

If the electric field within a conducting material is nonzero, it exerts a force on each of the mobile charges (usually electrons) and makes them move preferentially in a certain direction. With mobile charge in motion, the conductor cannot be in electrostatic equilibrium.
16.6 Conductors in Electrostatic Equilibrium

1. The electric field is zero at any point within a conducting material in electrostatic equilibrium. (If it weren’t, charges inside would move around – it would not be at equilibrium)

2. When a conductor is in electrostatic equilibrium, only its surface can have net charge. (Remember that if charge is on the surface, the field is zero inside? This is the flip side of that – the field can’t be zero inside unless the extra charge is on the surface. Look back at Example 16.8)

3. The electric field at the surface of the conductor is perpendicular to the surface. (If it were not, the charges would move around until it was.)
If there is charge $Q$ on a conducting sphere of radius $R$, we can use the results from before to find the field NEAR the surface. Since NEAR the surface, a sphere looks like an infinite flat surface (just look outside :)

$$E = 4\pi k Q/A = \frac{4\pi k Q}{4\pi R^2} = \frac{kQ}{R^2}$$

Smaller radius – larger field.
The surface charge density (charge per unit area) on a conductor in electrostatic equilibrium is highest at sharp points. The electric field is therefore greatest in those places as well.
For a conductor in electrostatic equilibrium,

5. There are no field lines within the conducting material (same as point #1).

6. Field lines that start or stop on the surface of a conductor are perpendicular to the surface where they intersect it (same as point #3).

7. The electric field just outside the surface of a conductor is strongest near sharp points.
Example 16.11

A solid conducting sphere that carries a total charge of \(-16 \, \mu\text{C}\) is placed at the center of a hollow conducting spherical shell that carries a total charge of \(+8 \, \mu\text{C}\). The conductors are in electrostatic equilibrium.

Determine the charge on the outer and inner surfaces of the shell and sketch a field line diagram.

Strategy.

We can apply any of the conclusions we just reached about conductors in electrostatic equilibrium as well as the properties of electric field lines.
Example 16.11 Solution 1

Field lines outside the solid sphere:

Field lines inside the shell:
Example 16.11 Solution 2

Complete field line sketch:
Application: Lightning Rods

Lightning rods (invented by Franklin) are often found on the roofs of tall buildings and old farmhouses.

The rod comes to a sharp point at the top. When a thunderstorm attracts charge to the top of the rod, the strong electric field at the point ionizes nearby air molecules, allowing charge to leak gently off instead of building up to a large value.

If the rod did not come to a sharp point, the electric field might not be large enough to ionize the air.
Lightning rod: Standing in a field

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Lightning rod: Standing in a field
(you become a lightning rod)
Application: Electrostatic Precipitator

Airflow

Dust collects on negative plates

Needle-like projections on positive plates
Gauss’s “law”, named after German mathematician Karl Friedrich Gauss (1777–1855), is a powerful statement of properties of the electric field. It relates the electric field on a closed surface—any closed surface—to the net charge inside the surface. Equivalent to Coulomb’s “law” (almost...)

A closed surface encloses a volume of space, so that there is an inside and an outside.

Gauss’s “law” says: I can tell you how much charge you have inside that “box” without looking inside; I’ll just look at the field lines that enter or exit the box.
If a box has no charge inside of it, then the same number of field lines that go into the box must come back out; there is nowhere for field lines to end or to begin.

If there is net positive charge inside, then there will be field lines starting on the positive charge that leave the box; then more field lines come out than go in.

If there is net negative charge inside, some field lines that go in end on the negative charge; more field lines go in than come out.
In order for Gauss’s “law” to be useful, we formulate it mathematically so that numbers of field lines are not involved.

To reformulate the “law”, there are two conditions to satisfy.

First, a mathematical quantity must be found that is proportional to the number of field lines leaving a closed surface.

Second, a proportionality must be turned into an equation by solving for the constant of proportionality.
16.7 Gauss’s “Law” for Electric Fields

The electric field is proportional to the number of field lines *per unit cross-sectional area*:

\[ E \propto \frac{\text{number of lines}}{\text{area}} \]
In general, the number of field lines crossing a surface is proportional to the *perpendicular component* of the field times the area:

\[ \text{number of lines} \propto E_\perp A = EA \cos \theta \]
The mathematical quantity that is proportional to the number of field lines crossing a surface is called the **flux of the electric field** (symbol $\Phi_E$; $\Phi$ is the Greek capital phi).

**Definition of Flux.**

\[
\Phi_E = E_\perp A = E_\perp A = EA \cos \theta
\]

For a closed surface, flux is defined to be positive if more field lines leave the surface than enter, or negative if more lines enter than leave. Flux is then positive if the net enclosed charge is positive and it is negative if the net enclosed charge is negative.
Since the net number of field lines is proportional to the net charge inside a closed surface, Gauss’s “law” takes the form.

\[ \Phi_E = \text{constant} \times q \]

where \( q \) stands for the net charge enclosed by the surface.

In Example 16.12 (and Problem 74), you can show that the constant of proportionality is \( 4\pi k = 1/\varepsilon_0 \).
Example 16.12

What is the flux through a sphere of radius \( r = 5.0 \text{ cm} \) that has a point charge \( q = -2.0 \mu \text{C} \) at its center?

Strategy.

In this case, there are two ways to find the flux. The electric field is known from Coulomb’s “law” and can be used to find the flux, or we can use Gauss’s “law”.
Example 16.12 Solution

\[ E = \frac{kq}{r^2} \]

\[ \theta = 0 \text{ everywhere} \]

\[ \Phi_E = EA = \frac{kq}{r^2} \times 4\pi r^2 = 4\pi kq \]

\[ \Phi_E = 4\pi kq \]

\[ = 4\pi \times 9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2} \times (-2.0 \times 10^{-6} \ C) \]

\[ = -2.3 \times 10^5 \ \frac{N \cdot m^2}{C^2} \]
Using Gauss’s “Law” to Find the Electric Field

As presented so far, Gauss’s “law” is a way to determine how much charge is inside a closed surface given the electric field on the surface, but it is more often used to find the electric field due to a distribution of charges.

When there are large numbers of charges, it is simpler to view the charge as a continuous distribution.
Charge Density

For a continuous distribution, the **charge density** is usually the most convenient way to describe how much charge is present. There are three kinds of charge densities:

- If the charge is spread throughout a volume, the relevant charge density is the charge per unit *volume* (symbol $\rho$).
- If the charge is spread over a two-dimensional surface, then the charge density is the charge per unit *area* (symbol $\sigma$).
- If the charge is spread over a one-dimensional line or curve, the appropriate charge density is the charge per unit *length* (symbol $\lambda$).
Example 16.13

Charge is spread \textit{uniformly} along a long thin wire. The charge per unit length on the wire is \( \lambda \) and is constant.

Find the electric field at a distance \( r \) from the wire, far from either end of the wire.
Example 16.13 Strategy

When concerned only with points near the wire, and far from either end, an approximately correct answer is obtained by assuming the wire is *infinitely long*.

How is it a simplification to *add* more charges? When using Gauss’s “law”, a symmetrical situation is far simpler than a situation that lacks symmetry.

An infinitely long wire with a uniform linear charge density has *axial symmetry*. 
Example 16.13 Solution 1

Correct

(a)

Incorrect

(b) (c)
Example 16.13 Solution 2

How much charge is enclosed by this cylinder?
Example 16.13 Solution 3

\[ \Phi_E = E_r A \]

\[ A = 2\pi r L \]

\[ q = \lambda L \]

\[ 4\pi k q = \Phi_E = E_r A \]

\[ E_r \left(2\pi r L\right) = 4\pi k \lambda L \]

\[ E_r = \frac{2k \lambda}{r} \]

The field direction is radially outward for \( \lambda > 0 \) and radially inward for \( \lambda < 0 \).
Example: Long metal pipe

Charge is spread *uniformly* along a long cylindrical pipe. The charge per unit length on the pipe is \( \lambda \) and is constant.

Find the electric field at a distance \( r \) from center of the pipe, far from either end of the pipe.

Solution is the SAME outside the pipe: 

\[
E_r = \frac{2k\lambda}{r}
\]
Example: metal pipe INSIDE Solution

\[ \Phi_E = E_r \cdot A \]

\[ A = 2\pi r L \]

\[ q = 0 ! \]

\[ 4\pi k q = \Phi_E = E_r \cdot A \]

\[ E_r (2\pi r L) = 0 \]

\[ E_r = 0 \]

So the ions leaving your engine feel NO force UNTIL they leave the tailpipe at the end. THEN they are attracted to the + charge on the car. When the charge on the car builds up enough, the force will be strong enough to PULL the ions back.
Example: metal pipe INSIDE

\[ E_r = 0 \]

So the ions leaving your engine feel NO force UNTIL they leave the tailpipe at the end. THEN they are attracted to the + charge on the car. When the charge on the car builds up enough, the force will be strong enough to PULL the ions back. This balance will mean much less charge on the car than we calculated earlier.
Chapter 16: Electric Forces and Fields

16.1 Electric Charge.
16.2 Electric Conductors and Insulators.
16.3 Coulomb’s “law”.
16.4 The Electric Field.
16.5 Motion of a Point Charge in a Uniform Electric Field.
16.6 Conductors in Electrostatic Equilibrium.
16.7 Gauss’s “law” for Electric Fields.
16.1 Electric Charge

Until the invention of the battery (Alessandro Volta) electricity experiments were basically static electricity, like when your clothes cling out of the dryer or you shock yourself after walking across a carpet.

One early worker was Benjamin Franklin (1750s). In his book on electricity, he describes:

• killing turkeys
• Proposes execution
• Knocked down up to 6 men holding hands (by essentially building up charge by rubbing)
A fluid model of electricity proposed that when you rub, you move charge FROM one object TO another, so the NET amount was fixed. (Franklin, et al., proposed ONLY one fluid).

So as you rubbed objects together, there was a DEFICIT of charge on one and an EXCESS on the other:
16.1 Electric Charge

“When a physicist makes a 50/50 guess, they’ll be wrong about 90% of the time” Dr Wm Deering, UNT

MUCH later experiments showed a need for TWO kinds of charge

J J Thomson (1897) showed there was a negatively charged particle called the electron.
Elementary Charge

The magnitude of charge on the proton and electron is the same. That amount of charge is called the elementary charge (symbol $e$).

$$e = 1.602 \times 10^{-19} \text{ C}$$

The net charge of any object is an integral multiple of the elementary charge.
### Proton, Electron, and Neutron

**Table 16.1** Masses and Electric Charges of the Proton, Electron, and Neutron.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass</th>
<th>Electric Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>$m_p = 1.673 \times 10^{-27}$ kg</td>
<td>$q_p = +e = +1.602 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Electron</td>
<td>$m_e = 9.109 \times 10^{-31}$ kg</td>
<td>$q_e = -e = -1.602 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Neutron</td>
<td>$m_n = 1.675 \times 10^{-27}$ kg</td>
<td>$q_n = 0$</td>
</tr>
</tbody>
</table>

Note that the mass of the electron is MUCH less – that is one reason the electrons usually are the charge that moves around – it’s just easier to move. (Another is that electrons are bound by the electric force and protons are bound by the MUCH STRONGER nuclear force).
Proton, Electron, and Neutron

Table 16.1 Masses and Electric Charges of the Proton, Electron, and Neutron.

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Protons and neutrons are NOT fundamental, but are made of three quarks, each charged at ± e/3 or ± 2e/3. So the proton is could be made up of three +e/3 quarks – so the charge is +e/3 + e/3 + e/3 = +e:

\[-\text{Are there other ways to arrange three quarks to get a total charge of } +e?\]

How about the neutron? How many ways are there to combine three quarks to get a net charge of zero?

(Clearly you can google this – and if that’s all you ever do, you’ll be well prepared for a career as a “googler” – I wonder how many of those there are?)
Proton, Electron, and Neutron

This was discovered by firing a beam of electrons at a proton and looking at how they bounced off of the proton.
Example 16.1

The magnitude of charge transferred when you walk across a carpet, reach out to shake hands, and unintentionally give a shock to a friend might be typically about 1 nC.

(a) If the charge is transferred by electrons only, how many electrons are transferred?

(b) If your body has a net charge of $-1 \text{ nC}$, estimate the percentage of excess electrons.

[Hint: The mass of the electron is only about 1/2000 that of a nucleon, so most of the mass of the body is in the nucleons. For an order-of-magnitude calculation, we can just assume that half of the nucleons are protons and half are neutrons. Look at the periodic table...].
Example 16.1 Strategy

Since the coulomb (C) is the SI unit of charge, the “n” must be the prefix “nano-” \( ( = 10^{-9} ) \). We know the value of the elementary charge in coulombs.

For part (b), we first make an order-of-magnitude estimate of the number of electrons in the human body.
Example 16.1 Solution

\[
\frac{-1 \times 10^{-9} \text{C}}{-1.6 \times 10^{-19} \text{C per electron}} = 6 \times 10^9 \text{ electrons}
\]

\[
\text{number of nucleons} = \frac{\text{mass of body}}{\text{mass per nucleon}} = \frac{70 \text{ kg}}{1.7 \times 10^{-27} \text{ kg}} = 4 \times 10^{28} \text{ nucleons}
\]

\[
\text{number of protons} = \frac{1}{2} \times 4 \times 10^{28} = 2 \times 10^{28} \text{ protons}
\]

The percentage of excess electrons is then.

\[
\frac{6 \times 10^9}{2 \times 10^{28}} \times 100\% = \left(3 \times 10^{-17}\right)\%
\]
Example – charging by idling your engine

As your engine runs, it exhausts gasses out the tailpipe. Many are ions (SO$_{4}^{2-}$ or NO$_{3}^{-}$). Since charge is conserved, the car (and it’s contents) must then be positively charged. This is why you sometimes get shocked as you get out of a car and touch something (or VERY RARELY, ignite the gas when you fill up a car at a gas station). We can roughly estimate how fast a car charges while idling:

A full tank empties in about 30 hours of idling. And gas has a density of about 0.75 kg/L. So gas burns in an engine at a rate of:

\[
15\text{gal}/30\text{ hours} \times 1\text{ hr}/60\text{ min} \times 4\text{L}/\text{gal} \times 0.75\text{ kg/L}
\]
Example – charging by idling your engine

15gal/30 hours x 1 hr/60 min x 4L/gal x 0.75 kg/L
= 25 g/min

EPA requires less than 10 ppm of sulfur in gasoline, so less than 250 \( \mu\)g/min of sulfur leaving the tailpipe. Using 32 g/mol of sulfur:

\[
\begin{align*}
250 \ \mu\text{g/min} & \times 1 \ \text{mole/32 g} \\
& \times 6 \times 10^{23}/\text{mole} \\
& \times 2 \times 1.6 \times 10^{-19} \ \text{C} \\
& = 1.5 \ \text{C/minute}
\end{align*}
\]

This implies that in a 10 minute drive, the car charges to about 15 C – we’ll find that this is a large overestimate.

(But gas pump fires are NOT caused by young people with phones….)
**Attraction, Repulsion, and Polarization**

Like charges repel one another; unlike charges attract one another.

**Polarization.**

An electrically neutral object may have regions of positive and negative charge within it, separated from one another. Such an object is **polarized**.

A polarized object can experience an electric force even though its net charge is zero.
Polarization by Induction

The net force on the paper is always attractive, regardless of the sign of charge on the rod. In this case, we say that the paper is polarized by induction.
Polarized Molecules

Some molecules (e.g., water) are intrinsically polarized.
Application: Hydrogen Bonds in Water

0.27 nm

Hydrogen bond
Application: Hydrogen Bonds in Water

Due largely to hydrogen bonding, water:
• is a liquid rather than a gas at room temperature;
• has a large specific heat;
• has a large heat of vaporization;
• is less dense as a solid (ice) than as a liquid;
• has a large surface tension;
• exhibits strong adhesive forces with some surfaces;
• is a powerful solvent of polar molecules.
Application: Hydrogen Bonds in DNA, RNA, and Proteins

Adenine

Thymine

Hydrogen bonds
16.2 Electric Conductors and Insulators

The difference is in HOW the atoms bond (chemistry) because ALL are the same inside – almost exactly as many electrons as protons. Difference is whether a current (collective motion of charge) can be generated.

Materials which support a current are called electric **conductors**, whereas materials which don’t are called electrical **insulators**.

Intermediate between conductors and insulators are the **semiconductors**.
Charging Insulators by Rubbing

When different insulating objects are rubbed against one another, both electrons and ions (charged atoms) can be transferred from one object to the other. (Difference in electron affinity – chemistry – makes charge move from one to the other).
Charging a Conductor by Contact

Touch a charged insulator to the conductor. Since the charge transferred to the conductor spreads out, the process can be repeated to build up more and more charge on the conductor.
Grounding a Conductor

How can a conductor be discharged? One way is to ground it.

Earth is a conductor because of the presence of ions and moisture, and it’s large enough that for many purposes it can be thought of as a limitless reservoir of charge.

To ground a conductor means to provide a conducting path between it and the Earth (or to another charge reservoir). A charged conductor that is grounded discharges because the charge spreads out by moving off the conductor and onto the Earth.
Charging a Conductor by Induction

A conductor is not necessarily discharged when it is grounded if there are other charges nearby.

It is even possible to charge an initially neutral conductor by grounding it.
Charging a Conductor by Induction

(a) Glass rod
   Silk cloth

(b) Metal sphere
   Insulating base

(c) Electrons flowing from ground through wire to sphere

(d) Disconnecting ground wire

(e) Equilibrium attained
Example 16.2

An electroscope is charged negatively and the gold foil leaves hang apart.

What happens to the leaves as the following operations are carried out in the order listed? Explain what you see after each step.

(a) You touch the metal bulb at the top of the electroscope with your hand.

(b) You bring a glass rod that has been rubbed with silk near the bulb without touching it. [Hint: A glass rod rubbed with silk is positively charged.].

(c) The glass rod touches the metal bulb.

[Diagram of an electroscope]
Example 16.2 Solution 1

(a) By touching the electroscope bulb with your hand, you ground it. Charge is transferred between your hand and the bulb until the bulb’s net charge is zero. Since the electroscope is now discharged, the foil leaves hang down.
(b) When the positively charged rod is held near the bulb, the electroscope becomes polarized by induction. Negatively charged free electrons are drawn toward the bulb, leaving the foil leaves with a positive net charge. The leaves hang apart due to the mutual repulsion of the net positive charges on them.
Example 16.2 Solution 3

(c) When the positively charged rod touches the bulb, some negative charge is transferred from the bulb to the rod.

The electroscope now has a positive net charge.

The glass rod still has a positive net charge that repels the positive charge on the electroscope, pushing it as far away as possible—toward the foil leaves.

The leaves hang farther apart, since they now have more positive charge on them than before.
Application: Photocopiers and Laser Printers
16.3 Coulomb’s “Law”

Coulomb’s “law” gives the electric force acting between two point charges.

A point charge is a point-like object with a nonzero electric charge.

Like gravity, the electric force is an inverse square “law” force.

Even for charges that are NOT points, Coulomb’s “law” gives a rough number for the force.
Magnitude of Electric Force

The magnitude of the electric force that each of two charges exerts on the other is given by:

\[ F = \frac{k|q_1||q_2|}{r^2} \]

The constant \( k \), which we call the Coulomb constant, can be written in terms of another constant \( \epsilon_0 \), the permittivity of free space:

\[ \epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \]

\[ F = \frac{|q_1||q_2|}{4\pi \epsilon_0 r^2} \]
Direction of Electric Force

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Problem-Solving Tips for Coulomb’s “Law” 1

1. Use consistent units; since we know $k$ in standard SI units $(N \cdot m^2/C^2)$, distances should be in meters and charges in coulombs. When the charge is given in $\mu C$ or nC, be sure to change the units to coulombs:

$$1 \, \mu C = 10^{-6} \, C \text{ and } 1 \, nC = 10^{-9} \, C.$$
Problem-Solving Tips for Coulomb’s “Law” 2

2. When finding the electric force on a single charge due to two or more other charges, find the force due to each of the other charges separately.

The net force on a particular charge is the vector sum of the forces acting on that charge due to each of the other charges.

Often it helps to separate the forces into $x$- and $y$-components, add the components separately, then find the magnitude and direction of the net force from its $x$- and $y$-components.
Problem-Solving Tips for Coulomb’s “Law” 3

3. If several charges lie along the same line, do not worry about an intermediate charge “shielding” the charge located on one side from the charge on the other side.

The electric force is long-range just as is gravity; the gravitational force on the Earth due to the Sun does not stop when the Moon passes between the two.
Example – force on two charged people.

We’ll find that a typical value for the charge you get (from walking across a carpet or sitting in a running car) is about 1 nC. What is the magnitude of the electric force they exert?

\[ F = \left(9 \cdot 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(10^{-9} \text{ C}\right) \left(10^{-9} \text{ C}\right)/(3 \text{ m})^2 \approx 10^{-9} \text{ N} \]

Compare this to the gravitational force they exert?

\[ F = \left(7 \cdot 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right) \left(70 \text{ kg}\right) \left(70 \text{ kg}\right)/(3 \text{ m})^2 \approx 4 \cdot 10^{-8} \text{ N} \]
Example 16.3

Suppose three point charges are arranged as follows. A charge $q_1 = +1.2 \ \mu\text{C}$ is located at the origin of an $(x, y)$ coordinate system; a second charge $q_2 = -0.60 \ \mu\text{C}$ is located at $(1.20 \text{ m}, 0.50 \text{ m})$ and the third charge $q_3 = +0.20 \ \mu\text{C}$ is located at $(1.20 \text{ m}, 0)$. What is the force on $q_2$ due to the other two charges?
Example 16.3 Strategy

The force on $q_2$ due to $q_1$ and the force on $q_2$ due to $q_3$ are determined separately.

After sketching a free-body diagram, we add the two forces as vectors.

Let the distance between charges 1 and 2 be $r_{12}$ and the distance between charges 2 and 3 be $r_{23}$. 
Example 16.3 Solution 1
Example 16.3 Solution 2

\[ r_{12} = \sqrt{r_{13}^2 + r_{23}^2} = 1.30 \text{ m} \]

\[
F_{21} = \frac{k|q_1||q_2|}{r_{12}^2} \\
= 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \left( 1.2 \times 10^{-6} \text{ C} \right) \times \left( 0.60 \times 10^{-6} \text{ C} \right) \left( 1.30 \text{ m} \right)^2 \\
= 3.83 \times 10^{-3} \text{ N} = 3.83 \text{ mN} \]
Example 16.3 Solution 3

\[ F_{23} = \frac{k|q_2||q_3|}{r_{23}^2} \]

\[ = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times \left( \frac{0.20 \times 10^{-6} \text{C}}{} \right) \times \left( \frac{0.60 \times 10^{-6} \text{C}}{} \right) \]

\[ = 4.32 \times 10^{-3} \text{N} = 4.32 \text{ mN} \]
Example 16.3 Solution 4

\[ F_{21x} = -F_{21} \sin \theta = -3.83 \, \text{mN} \times \frac{1.20 \, \text{m}}{1.30 \, \text{m}} = -3.53 \, \text{mN} \]

\[ F_{21y} = -F_{21} \cos \theta = -3.83 \, \text{mN} \times \frac{0.50 \, \text{m}}{1.30 \, \text{m}} = -1.47 \, \text{mN} \]

\[ F_{23x} = 0 \quad \text{and} \quad F_{23y} = -4.32 \, \text{mN} \]

\[ F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = 6.8 \, \text{mN} \]

\[ \phi = \tan^{-1} \left( \frac{3.53 \, \text{mN}}{5.79 \, \text{mN}} \right) = 31^\circ \]
Example 16.4

Two Styrofoam balls of mass 10.0 g are suspended by threads of length 25 cm. The balls are charged, after which they hang apart, each at $\theta = 15.0^\circ$ to the vertical.

(a) Are the signs of the charges the same or opposite?

(b) Are the magnitudes of the charges necessarily the same? Explain.

(c) Find the net charge on each ball, assuming that the charges are equal.
Example 16.4 Strategy

The situation is similar to the charged electroscope. Each ball exerts an electric force on the other since both are charged.

The *gravitational* forces that the balls exert on one another are negligibly small, but the gravitational forces that Earth exerts on the balls are not negligible.

The third force acting on each of the balls is due to the tension in a thread. We analyze the forces acting on a ball using an FBD. The sum of the three forces must be zero since the ball is in equilibrium.
Example 16.4 Solution 1

(a) The electric force is clearly repulsive—the balls are pushed apart—so the charges must have the same sign. There is no way to tell whether they are both positive or both negative.

(b) The force on either of the balls is proportional to the product of the two charge magnitudes; $F \propto q_1 q_2$. In accordance with Newton’s third “law”, Coulomb’s “law” says that the two forces that make up the interaction are equal in magnitude and opposite in direction. The charges are not necessarily equal.
Example 16.4 Solution 2

(c) \[ \sum F_x = F_E - T \sin \theta = 0 \]
\[ \sum F_y = T \cos \theta - mg = 0 \]

\[ F_E = T \sin \theta = \left( \frac{mg}{\cos \theta} \right) \sin \theta = mg \tan \theta \]

\[ F_E = \frac{k |q|^2}{r^2} \quad r = 2 \left( d \sin \theta \right) \quad d = 25 \text{ cm} \]

\[ |q|^2 = \frac{F_E r^2}{k} \]
Example 16.4 Solution 3

(c) continued.

\[ |q|^2 = \frac{F_E r^2}{k} \]

\[ |q|^2 = \frac{(mg \tan \theta)(2d \sin \theta)^2}{k} \]

\[ = \frac{4d^2 mg \tan \theta \sin^2 \theta}{k} \]

\[ |q| = \sqrt{\frac{4 \times (0.25 \text{ m})^2 \times 0.0100 \text{ kg} \times 9.8 \text{ N/kg} \times \tan 15.0^\circ \times \sin^2 15.0^\circ}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} \]

\[ = 0.22 \mu \text{C} \]

The charges can either both be positive or both negative, so the charges are either both +0.22 $\mu$C or both -0.22 $\mu$C.
16.4 The Electric Field

If a point charge \( q \) is in the vicinity of other charges, it experiences an electric force, \( \vec{F} \).

The **electric field** at any point is defined to be the electric force *per unit charge* at that point. The electric field is denoted. \( \vec{E} \).
16.4 The Electric Field

\[ \vec{E} = \frac{\vec{F}_E}{q} \]

The SI units of the electric field are N/C.

(Much like the gravitational field: \( \vec{g} = \frac{\vec{F}_{grav}}{m} \))

Why Define an Electric Field?

One reason is that once we know the electric field at some point, then it is easy to calculate the electric force on any point charge \( q \) placed there:

\[ \vec{F}_E = q \vec{E} \]
Why Define an Electric Field?

Another reason is that to calculate the FORCE we need both charges ($q_1$ and $q_2$); to calculate the field, we will see we only need one – useful when we design:

- antennas
- cell phones
- WiFi routers
- radar
- etc
Example 16.5

A small sphere of mass 5.10 g is hanging vertically from an insulating thread that is 12.0 cm long. By charging some nearby flat metal plates, the sphere is subjected to a horizontal electric field of magnitude $7.20 \times 10^5$ N/C. As a result, the sphere is displaced 6.00 cm horizontally in the direction of the electric field.

(a) What is the angle $\theta$ that the thread makes with the vertical?

(b) What is the tension in the thread?

(c) What is the charge on the sphere?
Example 16.5 Strategy

We assume that the sphere is small enough to be treated as a point charge.

Then the electric force on the sphere is given by $F_E = qE$

The figure shows that the sphere is pushed to the right by the field; therefore, the electric force is to the right. Since the electric field and force have the same direction, the charge on the sphere is positive.

After drawing an FBD showing all the forces acting on the sphere, we set the net force on the sphere equal to zero since it hangs in equilibrium.
Example 16.5 Solution 1

(a)

\[
\sin \theta = \frac{6.00 \text{ cm}}{12.0 \text{ cm}} = 0.500 \quad \text{and} \quad \theta = 30.0^\circ
\]
Example 16.5 Solution 2

(b)

\[ \sum F_y = T \cos \theta - mg = 0 \]

\[ T = \frac{mg}{\cos \theta} = \frac{5.10 \times 10^{-3} \text{kg} \times 9.80 \text{ N/kg}}{\cos 30.0^\circ} = 0.0577 \text{ N} \]
Example 16.5 Solution 3

(c) \[ \sum F_x = |q| E - T \sin \theta = 0 \]

\[ |q| = \frac{T \sin \theta}{E} = \frac{\left(5.77 \times 10^{-2} \text{ N}\right) \sin 30.0^\circ}{7.20 \times 10^5 \text{ N/C}} = 40.1 \text{ nC} \]

\[ q = 40.1 \text{ nC} \]
Electric Field due to a Point Charge

The electric field due to a single point charge $Q$ can be found using Coulomb’s “law”.

Imagine a positive test charge $q$ placed at various locations. Coulomb’s “law” says that the force acting on the test charge is.

$$ F = \frac{k q |Q|}{r^2} $$

The electric field strength is then.

$$ E = \frac{F}{|q|} = \frac{k |Q|}{r^2} $$
Principle of Superposition

The electric field at any point is the vector sum of the field vectors at that point caused by each charge separately.
Example 16.6

Two point charges are located on the $x$-axis. Charge $q_1 = +0.60 \ \mu\text{C}$ is located at $x = 0$; charge $q_2 = -0.50 \ \mu\text{C}$ is located at $x = 0.40 \ \text{m}$. Point $P$ is located at $x = 1.20 \ \text{m}$.

What is the magnitude and direction of the electric field at point $P$ due to the two charges?
Example 16.6 Strategy

We can determine the field at $P$ due to $q_1$ and the field at $P$ due to $q_2$ separately using Coulomb’s “law” and the definition of the electric field.

In each case, the electric field points in the direction of the electric force on a positive test charge at point $P$.

The sum of these two fields is the electric field at $P$. 
Example 16.6 Solution 1

\[ E_1 = \frac{k|q_1|}{r_1^2} \]

\[ = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2} \times \frac{0.60 \times 10^{-6} \, C}{(1.20 \, m)^2} \]

\[ = 3.75 \times 10^3 \, \text{N/C} \]
Example 16.6 Solution 2

\[ E_2 = \frac{k|q_2|}{r_2^2} \]
\[ = 8.99 \times 10^9 \frac{N\cdot m^2}{C^2} \times \frac{0.50 \times 10^{-6} \, C}{(0.80 \, m)^2} \]
\[ = 7.02 \times 10^3 \, \text{N/C} \]
Example 16.6 Solution 3

The electric field at $P$ is $3.3 \times 10^3$ N/C in the $-x$-direction.

$$E = 7.02 \times 10^3 \text{ N/C} = 3.75 \times 10^3 \text{ N/C} = 3.3 \times 10^3 \text{ N/C}$$
Example 16.7

Three point charges are placed at the corners of a rectangle, as shown in the figure.

(a) What is the electric field due to these three charges at the fourth corner, point $P$?

(b) What is the acceleration of an electron located at point $P$? Assume that no forces other than that due to the electric field act on it.
Example 16.7 Strategy

(a) After determining the magnitude and direction of the electric field at point $P$ due to each point charge individually, we use the principle of superposition to add them as vectors.

(b) Since we have already calculated the electric field at point $P$, the force on the electron is given by $\mathbf{F} = q\mathbf{E}$, where $q = -e$ is the charge of the electron.
Example 16.7 Solution 1

(a)

\[
E_1 = \frac{k|q_1|}{r_1^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m} \cdot \text{C}^{-2} \times 4.0 \times 10^{-6} \text{ C}}{(0.50 \text{ m})^2} = 1.44 \times 10^5 \times \text{N/C}
\]

A similar calculation with \(|q_3| = 1.0 \times 10^{-6} \text{ C}\) and \(r_3 = 0.20 \text{ m}\) yields \(E_3 = 2.25 \times 10^5 \text{ N/C}\). Using the Pythagorean theorem to find \(r_2 = \sqrt{(0.50 \text{ m})^2 + (0.20 \text{ m})^2}\), we have

\[
E_2 = \frac{kq_2}{r_2^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m} \cdot \text{C}^{-2} \times 6.0 \times 10^{-6} \text{ C}}{(0.50 \text{ m})^2 + (0.20 \text{ m})^2} = 1.86 \times 10^5 \text{ N/C}
\]
Example 16.7 Solution 2

(a) continued.

\[
\begin{align*}
\cos \theta &= \frac{r_1}{r_2} = 0.928 \\
\sin \theta &= 0.371
\end{align*}
\]

\[
\begin{align*}
\sum E_x &= E_{1x} + E_{2x} + E_{3x} = (-E_1) + (-E_2 \cos \theta) + 0 = -3.17 \times 10^5 \text{ N/C} \\
\sum E_y &= E_{1y} + E_{2y} + E_{3y} = 0 + E_2 \sin \theta - E_3 = -1.56 \times 10^5 \text{ N/C}
\end{align*}
\]

The magnitude of the electric field is then

\[
E = \sqrt{E_x^2 + E_y^2} = 3.5 \times 10^5 \text{ N/C}
\]

and the direction is at angle

\[
\phi = \tan^{-1} \left| \frac{E_y}{E_x} \right| = 26^\circ \text{ below the } -x \text{-axis.}
\]
Example 16.7 Solution 3

(b) The force on the electron is \( \mathbf{F} = q \mathbf{E} \). Its acceleration is then \( \mathbf{a} = q_e \mathbf{E}/m_e \). The electron charge \( q_e = -e \) and mass \( m_e \) are given in Table 16.1. The acceleration has magnitude \( a = eE/m_e = 6.2 \times 10^{16} \text{ m/s}^2 \). The direction of the acceleration is the direction of the electric force, which is opposite the direction of \( \mathbf{E} \) since the electron’s charge is negative.

Notice how CRAZY big the acceleration is – that’s not a misprint – electron accelerations are HUGE under very reasonable circumstances.
Electric Field Lines

It is often difficult to make a visual representation of an electric field using vector arrows; the vectors drawn at different points may overlap and become impossible to distinguish.

Another visual representation of the electric field is a sketch of the electric field lines, a set of continuous lines that represent both the magnitude and the direction of the electric field vector as follows. (Invented by Michael Faraday, who was a “visual learner”)
Interpretation of Electric Field Lines

The direction of the electric field vector at any point is *tangent to the field line* passing through that point and in the direction indicated by arrows on the field line.

The electric field is strong where field lines are close together and weak where they are far apart.
Rules for Sketching Field Lines:

Electric field lines can start only on positive charges and can end only on negative charges.

The number of lines starting on a positive charge (or ending on a negative charge) is proportional to the magnitude of the charge. (The total number of lines you draw is arbitrary; the more lines you draw, the better the representation of the field.)
Rules for Sketching Field Lines:

Field lines never cross. The electric field at any point has a unique direction; if field lines crossed, the field would have two directions at the same point.

Impossible

\[ \vec{E} = ? \]
Field Lines for a Point Charge

The figure shows sketches of the field lines due to single point charges. The field lines show that the direction of the field is radial (away from a positive charge or toward a negative charge).
Field Lines for a Point Charge

The lines are close together near the point charge, where the field is strong, and are more spread out farther from the point charge, showing that the field strength diminishes with distance.
A pair of point charges with equal and opposite charges that are near one another is called a dipole (literally two poles).

To find the electric field due to the dipole at various points by using Coulomb’s “law” would be extremely tedious, but sketching some field lines immediately gives an approximate idea of the electric field (next slide).
Electric Field due to a Dipole

\[ \vec{E} + \vec{E}_- - \vec{E}_+ \]
Example 16.8

A thin metallic spherical shell of radius $R$ carries a total charge $Q$, which is positive. The charge is spread out evenly over the shell’s outside surface. Sketch the electric field lines in two different views of the situation:

(a) the spherical shell is tiny, and you are looking at it from distant points;

(b) you are looking at the field inside the shell’s cavity.

In (a), also sketch $\mathbf{E}$ field vectors at two different points outside the shell.
Example 16.8 Strategy

Since the charge on the shell is positive, field lines begin on the shell.

A sphere is a highly symmetrical shape: standing at the center, it looks the same in any chosen direction. This symmetry helps in sketching the field lines.
Example 16.8 Solution 1

(a)
Example 16.8 Solution 2

(b)

We can see later that NO field in a spherical shell means the electric field is PRECISELY inverse square.
Example 16.8 Solution 2

(b) We can see later that NO field in a spherical shell means the electric field is PRECISELY inverse square.

\[ \vec{E} = \frac{kq}{r^2} \hat{r} \]

Is this EXACTLY 2 or just really close to 2?
16.5 Motion of a Point Charge in a Uniform Electric Field

The simplest example of how a charged object responds to an electric field is when the electric field (due to other charges) is **uniform**—that is, has the same magnitude and direction at every point.

\[
E = \frac{Q}{\varepsilon_0 A}
\]

\[
\vec{F} = q\vec{E}
\]

\[
\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}
\]
First, how to make a Uniform Electric Field

Imagine a large, flat plate carrying a positive charge $Q$, and consider the electric field at a point near the plate but NOT near any of the edges.

The NET field is AWAY from the plate and primarily due to charges that are “below” the point.
First, how to make a Uniform Electric Field

Farther from the plate, the field is weaker (inverse square) BUT MORE charge is “below” the point.

Since the electric force is inverse square \((1/r^2)\), these effects EXACTLY cancel. The field is UNIFORM.

!! What if the plate had a negative charge?
First, how to make a Uniform Electric Field

From units alone, the magnitude of the field must have things of the form:

\[ E = \{\text{no units}\} k \left[ \frac{\text{N} \text{ m}^2}{\text{C}^2} \right] \frac{\text{charge [Coulombs]}}{\text{distance squared [m}^2]} \]

Distance squared – Can’t be a distance (would change E)

Use AREA of plate.
First, how to make a Uniform Electric Field

So this:
\[ E = \{\text{no units}\} k \frac{[\text{N m}^2/\text{C}^2]}{[\text{charge [Coulombs]} / \text{distance squared [m}^2)]} \]

Becomes:
\[ E = \{2\pi\} k \frac{Q}{A} \]
First, how to make a Uniform Electric Field

Field outside is zero.
Field between is double.

\[ \vec{E}_+ = \vec{E}_- \]
\[ \vec{E} = 0 \]

\[ \vec{E} = 4\pi k \frac{Q}{A} \]

\[ \vec{E} = 0 \]
Acceleration of a Charged Particle due to an Electric Field

\[ a = \frac{\vec{F}}{m} \quad \frac{q\vec{E}}{m} \]

Another example of constant force is a weight near the Earth’s surface (like you did in mechanics!) so the behavior is the same.

Except one thing: The direction of the acceleration is either parallel (for a positive charge) or antiparallel (for a negative charge) to the electric field. (No such thing as negative mass...as far as we know...)
Example 16.9

A cathode ray tube (CRT) is used to accelerate electrons in some televisions, computer monitors, oscilloscopes, and x-ray tubes. Electrons from a heated filament pass through a hole in the cathode; they are then accelerated by an electric field between the cathode and the anode (next slide).

Suppose an electron passes through the hole in the cathode at a velocity of $1.0 \times 10^5$ m/s toward the anode. The electric field is uniform between the anode and cathode and has a magnitude of $1.0 \times 10^4$ N/C.

(a) What is the acceleration of the electron?

(b) If the anode and cathode are separated by 2.0 cm, what is the final velocity of the electron?
Example 16.9

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Example 16.9 Strategy

Given: initial speed $v_i = 1.0 \times 10^5$ m/s;
    separation between plates $d = 0.020$ m;
    electric field magnitude $E = 1.0 \times 10^4$ N/C.

Look up: electron mass $m_e = 9.109 \times 10^{-31}$ kg;
    electron charge $q = -e = -1.602 \times 10^{-19}$ C.

Find: (a) acceleration; (b) final velocity.
Example 16.9 Solution 1

(a) 
\[ F_g = mg = 9.109 \times 10^{-31} \text{ kg} \times 9.8 \text{ m/s}^2 = 8.9 \times 10^{-30} \text{ N} \]
\[ F_E = eE = 1.602 \times 10^{-19} \text{ C} \times 1.0 \times 10^4 \text{ N/C} = 1.6 \times 10^{-15} \text{ N} \]
\[ a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{1.602 \times 10^{-19} \text{ C} \times 1.0 \times 10^4 \text{ N/C}}{9.109 \times 10^{-31} \text{ kg}} = 1.76 \times 10^{15} \text{ m/s}^2 \]

To two significant figures, \( a = 1.8 \times 10^{15} \text{ m/s}^2 \). Since the charge on the electron is negative, the direction of the acceleration is opposite to the electric field, or to the right in the figure.
Example 16.9 Solution 2

(b)

\[ v_f = \sqrt{v_i^2 + 2ad} \]

\[ = \sqrt{(1.0 \times 10^5 \text{ m/s})^2 + 2 \times 1.76 \times 10^{15} \text{ m/s}^2 \times 0.020 \text{ m}} \]

\[ = 8.4 \times 10^6 \text{ m/s to the right} \]
Example 16.10

An electron is projected horizontally into the uniform electric field directed vertically downward between two parallel plates. The plates are 2.00 cm apart and are of length 4.00 cm. The initial speed of the electron is \(v_i = 8.00 \times 10^6\) m/s. As it enters the region between the plates, the electron is midway between the two plates; as it leaves, the electron just misses the upper plate.

What is the magnitude of the electric field?
Example 16.10 Strategy

Given: \( \Delta x = 4.00 \) cm; \( \Delta y = 1.00 \) cm; \( v_x = 8.00 \times 10^6 \) m/s

Find: electric field strength, \( E \).
Example 16.10 Solution

$$\Delta t = \frac{\Delta x}{v_x} = \frac{4.00 \times 10^{-2} \text{ m}}{8.00 \times 10^{14} \text{ m/s}} = 5.00 \times 10^{-9} \text{ s}$$

$$\Delta y = \frac{1}{2} a_y \left(\Delta t\right)^2$$

$$a_y = \frac{2 \Delta y}{\left(\Delta t\right)^2} = \frac{2 \times 1.00 \times 10^{-2} \text{ m}}{\left(5.00 \times 10^{-9} \text{ s}\right)^2} = 8.00 \times 10^{14} \text{ m/s}^2$$

$$F_y = qE = m_e a_y$$

$$E_y = \frac{m_e a_y}{q} = \frac{9.109 \times 10^{-31} \text{ kg} \times 8.00 \times 10^{14} \text{ m/s}^2}{6.602 \times 10^{-19} \text{ C}} = -4.55 \times 10^3 \text{ N/C}$$

Since the field has no x-component, its magnitude is $4.55 \times 10^3 \text{ N/C}$. 
EXAMPLE: Photoelectric effect

Heinrich Hertz discovered that electrons fly off a metal plate faster or slower according to the color of light you shine on the metal plate. How did he measure the speed?

\[ E = 4\pi k \frac{Q}{A} \]

Detect electrons leaving through the hole and adjust the field between the plates until they no longer leave (final speed is zero).
EXAMPLE: Photoelectric effect

\[ v^2 = v_0^2 + 2a(x - x_0) \]

Use final \( v = 0 \)

\[ v_0 = \sqrt{-2a(x - x_0)} = \sqrt{-2ad} \]

Use Newton’s Second equation:

\[ a = \frac{F}{m} = \frac{q}{m}E = \frac{q}{m}\frac{Q}{\epsilon_0 A} \]

\[ v_0 = \sqrt{-2 \left( \frac{q}{m} \right) \left( \frac{Q}{\epsilon_0 A} \right) d} \]
The most easily polarized materials are conductors because they contain highly mobile charges that can move freely through the entire volume of the material.

It is useful to examine the distribution of charge in a conductor, whether the conductor has a net charge or lies in an externally applied field, or both.

We restrict our attention to a conductor in which the mobile charges are at rest in equilibrium, a situation called **electrostatic equilibrium**.
16.6 Conductors in Electrostatic Equilibrium

If charge is put on a conductor, mobile charges move about until a stable distribution is attained. The same thing happens when an external field is applied or changed—charges move in response to the external field, but they **soon** eventually reach an equilibrium distribution.

If the electric field within a conducting material is nonzero, it exerts a force on each of the mobile charges (usually electrons) and makes them move preferentially in a certain direction. With mobile charge in motion, the conductor cannot be in electrostatic equilibrium.
16.6 Conductors in Electrostatic Equilibrium

1. The electric field is zero at any point within a conducting material in electrostatic equilibrium. (If it weren’t, charges inside would move around – it would not be at equilibrium)

2. When a conductor is in electrostatic equilibrium, only its surface can have net charge. (Remember that if charge is on the surface, the field is zero inside? This is the flip side of that – the field can’t be zero inside unless the extra charge is on the surface. Look back at Example 16.8)

3. The electric field at the surface of the conductor is perpendicular to the surface. (If it were not, the charges would move around until it was.)
16.6 Conductors in Electrostatic Equilibrium

If there is charge $Q$ on a conducting sphere of radius $R$, we can use the results from before to find the field NEAR the surface. Since NEAR the surface, a sphere looks like an infinite flat surface (just look outside :)

$$E = \frac{4 \pi k Q}{A} = \frac{4 \pi k Q}{4 \pi R^2} = \frac{kQ}{R^2}$$

Smaller radius – larger field.
16.6 Conductors in Electrostatic Equilibrium

The surface charge density (charge per unit area) on a conductor in electrostatic equilibrium is highest at sharp points. The electric field is therefore greatest in those places as well.
For a conductor in electrostatic equilibrium,

5. There are no field lines within the conducting material (same as point #1).

6. Field lines that start or stop on the surface of a conductor are perpendicular to the surface where they intersect it (same as point #3).

7. The electric field just outside the surface of a conductor is strongest near sharp points.
Example 16.11

A solid conducting sphere that carries a total charge of \(-16 \, \mu\text{C}\) is placed at the center of a hollow conducting spherical shell that carries a total charge of \(+8 \, \mu\text{C}\). The conductors are in electrostatic equilibrium.

Determine the charge on the outer and inner surfaces of the shell and sketch a field line diagram.

Strategy.

We can apply any of the conclusions we just reached about conductors in electrostatic equilibrium as well as the properties of electric field lines.
Example 16.11 Solution 1

Field lines outside the solid sphere:

Field lines inside the shell:
Example 16.11 Solution 2

Complete field line sketch:
Application: Lightning Rods

Lightning rods (invented by Franklin) are often found on the roofs of tall buildings and old farmhouses.

The rod comes to a sharp point at the top. When a thunderstorm attracts charge to the top of the rod, the strong electric field at the point ionizes nearby air molecules, allowing charge to leak gently off instead of building up to a large value.

If the rod did not come to a sharp point, the electric field might not be large enough to ionize the air.
Lightning rod: Standing in a field

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Lightning rod: Standing in a field
(you become a lightning rod)
Application: Electrostatic Precipitator

Airflow

Dust collects on negative plates

Needle-like projections on positive plates
16.7 Gauss’s “Law” for Electric Fields

Gauss’s “law”, named after German mathematician Karl Friedrich Gauss (1777–1855), is a powerful statement of properties of the electric field. It relates the electric field on a closed surface—*any* closed surface—to the net charge inside the surface. Equivalent to Coulomb’s “law” (almost...)

A **closed surface** encloses a volume of space, so that there is an inside and an outside.

Gauss’s “law” says: I can tell you how much charge you have inside that “box” without looking inside; I’ll just look at the field lines that enter or exit the box.
16.7 Gauss’s “Law” for Electric Fields

If a box has no charge inside of it, then the same number of field lines that go into the box must come back out; there is nowhere for field lines to end or to begin.

If there is net positive charge inside, then there will be field lines starting on the positive charge that leave the box; then more field lines come out than go in.

If there is net negative charge inside, some field lines that go in end on the negative charge; more field lines go in than come out.
16.7 Gauss’s “Law” for Electric Fields

In order for Gauss’s “law” to be useful, we formulate it mathematically so that numbers of field lines are not involved.

To reformulate the “law”, there are two conditions to satisfy.

First, a mathematical quantity must be found that is proportional to the number of field lines leaving a closed surface.

Second, a proportionality must be turned into an equation by solving for the constant of proportionality.
16.7 Gauss’s “Law” for Electric Fields.

The electric field is proportional to the number of field lines per unit cross-sectional area:

\[ E \propto \frac{\text{number of lines}}{\text{area}} \]
In general, the number of field lines crossing a surface is proportional to the perpendicular component of the field times the area:

\[
\text{number of lines} \propto E \perp A = EA \cos \theta
\]
Electric Flux

The mathematical quantity that is proportional to the number of field lines crossing a surface is called the flux of the electric field (symbol $\Phi_E$; $\Phi$ is the Greek capital phi).

Definition of Flux.

$$\Phi_E = E \cdot A = E \cdot A_{\perp} = E A \cos \theta$$

For a closed surface, flux is defined to be positive if more field lines leave the surface than enter, or negative if more lines enter than leave. Flux is then positive if the net enclosed charge is positive and it is negative if the net enclosed charge is negative.
16.7 Gauss’s “Law” for Electric Fields.

Since the net number of field lines is proportional to the net charge inside a closed surface, Gauss’s “law” takes the form.

$$\Phi_E = \text{constant} \times q$$

where $q$ stands for the net charge enclosed by the surface.

In Example 16.12 (and Problem 74), you can show that the constant of proportionality is $4\pi k = 1/\epsilon_0$.

Gauss’s “law”.

$$\Phi_E = 4\pi k q = q/\epsilon_0$$
Example 16.12

What is the flux through a sphere of radius $r = 5.0$ cm that has a point charge $q = -2.0$ $\mu$C at its center?

Strategy.
In this case, there are two ways to find the flux. The electric field is known from Coulomb’s “law” and can be used to find the flux, or we can use Gauss’s “law”.

Example 16.12 Solution

\[ E = \frac{kq}{r^2} \]

\[ \theta = 0 \text{ everywhere} \]

\[ \Phi_E = E A = \frac{kq}{r^2} \times 4\pi r^2 = 4\pi kq \]

\[ \Phi_E = 4\pi kq \]

\[ = 4\pi \times 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \times (-2.0 \times 10^{-6} \text{ C}) \]

\[ = -2.3 \times 10^5 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \]
Using Gauss’s “Law” to Find the Electric Field

As presented so far, Gauss’s “law” is a way to determine how much charge is inside a closed surface given the electric field on the surface, but it is more often used to find the electric field due to a distribution of charges.

When there are large numbers of charges, it is simpler to view the charge as a continuous distribution.
**Charge Density**

For a continuous distribution, the **charge density** is usually the most convenient way to describe how much charge is present. There are three kinds of charge densities:

- If the charge is spread throughout a volume, the relevant charge density is the charge per unit *volume* (symbol $\rho$).
- If the charge is spread over a two-dimensional surface, then the charge density is the charge per unit *area* (symbol $\sigma$).
- If the charge is spread over a one-dimensional line or curve, the appropriate charge density is the charge per unit *length* (symbol $\lambda$).
Example 16.13

Charge is spread \textit{uniformly} along a long thin wire. The charge per unit length on the wire is $\lambda$ and is constant.

Find the electric field at a distance $r$ from the wire, far from either end of the wire.
Example 16.13 Strategy

When concerned only with points near the wire, and far from either end, an approximately correct answer is obtained by assuming the wire is *infinitely long*.

How is it a simplification to *add* more charges? When using Gauss’s “law”, a symmetrical situation is far simpler than a situation that lacks symmetry.

An infinitely long wire with a uniform linear charge density has *axial symmetry*. 
Example 16.13 Solution 1

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Correct

Incorrect

(a)

(b)

(c)
Example 16.13 Solution 2

How much charge is enclosed by this cylinder?
Example 16.13 Solution 3

\[ \Phi_E = E_r A \]

\[ A = 2\pi r L \]

\[ q = \lambda L \]

\[ 4\pi k q = \Phi_E = E_r A \]

\[ E_r \left(2\pi r L\right) = 4\pi k \lambda L \]

\[ E_r = \frac{2k \lambda}{r} \]

The field direction is radially outward for \( \lambda > 0 \) and radially inward for \( \lambda < 0 \).
Example: Long metal pipe

Charge is spread *uniformly* along a long cylindrical pipe. The charge per unit length on the pipe is \( \lambda \) and is constant.

Find the electric field at a distance \( r \) from center of the pipe, far from either end of the pipe.

Solution is the SAME outside the pipe: \( E_r = \frac{2k\lambda}{r} \)
Example: metal pipe INSIDE Solution

\[ \Phi_E = E_r A \]

\[ A = 2\pi r L \]

\[ q=0 \]

\[ 4\pi k q = \Phi_E = E_r A \]

\[ E_r (2\pi r L) = 0 \]

\[ E_r = 0 \]

So the ions leaving your engine feel NO force UNTIL they leave the tailpipe at the end. THEN they are attracted to the + charge on the car. When the charge on the car builds up enough, the force will be strong enough to PULL the ions back.
Example: metal pipe INSIDE

\[ E_r = 0 \]

So the ions leaving your engine feel NO force UNTIL they leave the tailpipe at the end. THEN they are attracted to the + charge on the car. When the charge on the car builds up enough, the force will be strong enough to PULL the ions back. This balance will mean much less charge on the car than we calculated earlier.