Two charges, $q_1$ and $q_2$, are placed on the $x$-axis, one at $x=0$ and one at $x=a$. Where on the $x$-axis is the electric field zero?

First, assume both are positive. (You should consider the other possibilities — what if both are negative? What if anything changes? or if they are opposite signs?)

Then we know the field is never zero for $x<0$ or $x>a$. [Make sure you understand why]
So our answer must be \(0 < x < a\).
(This will be useful later).

So we write the electric field in this region:

\[
E_{\text{net}} = E_1 + E_2
\]

\[
= \frac{kq_1}{x^2} - \frac{kq_2}{(a-x)^2}
\]

Now impose \(E_{\text{net}} = 0\) and solve for \(x\):

\[
k \left[ \frac{q_1}{x^2} - \frac{q_2}{(a-x)^2} \right] = 0
\]

\[
\frac{q_1}{x^2} = \frac{q_2}{(a-x)^2}
\]

\[
(a-x)^2 = \frac{q_2}{q_1} x^2
\]

To make this easier to write;
Define \( \alpha^2 = \frac{q_1}{q_1} \)

\[
a^2 - 2ax + x^2 = \alpha^2 x^2
\]

\[
(1 - \alpha^2)x^2 - 2ax + a^2 = 0
\]

Use the quadratic formula:

\[
x = \frac{2a \pm \sqrt{(2a)^2 - 4(1 - \alpha^2)a^2}}{2(1 - \alpha^2)}
\]

\[
= \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4\alpha^2 a^2}}{2(1 - \alpha^2)}
\]

\[
= \frac{2a \pm 2\alpha x}{2(1 - \alpha^2)}
\]

\[
= a \cdot \frac{1 \pm \alpha}{1 - \alpha^2}
\]
Recall that only $0 < x < a$ are possible answers.
So the fraction:
\[
\frac{1 \pm x}{1 - x^2}
\]

must be less than 1.
This is only possible if we use the minus sign in the numerator. Also, note that $(1 - x^2) = (1 - x)(1 + x)$.
So \[ x = a \frac{(1 - x)}{(1 - x)(1 + x)} = \frac{a}{1 + x} \]

Put in the numbers from 16.49 and see if you get the same answer. What if both charges are doubled? What if $q_1 = q_2$?