EXAM III PHYS 192 18 NOV 2021  D. Norwood

Some stuff to think about: Read all the questions first and do the ones you find easier first. Feel free to ask if something is unclear or you feel you need other information - I won't tell you how to work the problem, but I'm happy to clarify. TELL ME WHO YOU ARE. Above all, relax and say your mantra. Champagne for my real friends.

1. (25 points) A helium atom with only one electron (a "singly ionized" helium atom) is kind of like a hydrogen atom (a point positive charge at the center with a single electron orbiting) but with two protons at the center instead of one (and one or two neutrons that don't matter for this problem). Because the two protons pull harder on the electron, it orbits closer: \( r = 2.5 \times 10^{-11} \text{ m} \) and faster: \( v = 4 \times 10^6 \text{ m/s} \) and so generates a higher current as it orbits: \( I = \frac{qv}{2\pi r} = 4 \text{ mA} \).

A) Calculate the strength of the magnetic field created by the electron at the center of the orbit (where the nucleus is).

\[
B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ N/A}^2)(4 \times 10^{-3} \text{ A})}{2(2.5 \times 10^{-11} \text{ m})} = 100 \frac{\text{N}}{\text{A} \cdot \text{m}}
\]

B) Each of the protons can be considered as charge of \( +1.6 \times 10^{-5} \text{ C} \) orbiting in a circle of radius \( r = 1 \times 10^{-15} \text{ m} \) at speed \( v = 3 \times 10^7 \text{ m/s} \) (so creating a current of about \( 765 \text{ A} \)). Calculate the magnetic moment of each of these protons.

\[
\mu = IA = (765 \text{ A})(\pi)(10^{-15} \text{ m})^2
= 2.4 \times 10^{-27} \text{ A} \cdot \text{m}^2
\]

C) Calculate the maximum torque the orbiting electron can exert on one of the protons.

\[
\tau = \mu B = (2.4 \times 10^{-27} \text{ A} \cdot \text{m}^2)(100 \frac{\text{N}}{\text{A} \cdot \text{m}})
= 2.4 \times 10^{-25} \text{ N} \cdot \text{m}
\]
2. (15 points) The figure shows a wire carrying a constant current of 1A and a circular loop 20 cm away with radius 1 cm and resistance, R. The circular loop is flipped over (so oriented in the plane of the wire, as shown, rotated to 90° away from the plane and then flat again – ask if you're unsure).

A) Sketch (and label) a graph of the magnetic flux through the loop as it flips over.

B) On the same graph, sketch and label the induced current. Indicate the direction (CW or CCW) of the induced current on the graph.
2. **(15 points)** The figure shows a wire carrying a constant current of 1 A and a circular loop 20 cm away with radius 1 cm and resistance, R. The circular loop is flipped over (so oriented in the plane of the wire, as shown, rotated to 90° away from the plane and then flat again – ask if you’re unsure).

A) Sketch (and label) a graph of the magnetic flux through the loop as it flips over.

B) On the same graph, sketch and label the induced current. Indicate the direction (CW or CCW) of the induced current on the graph.
3. (20 points) A coil has an inductance of 0.250 H and a resistance of 50.0 Ω. The coil is connected to a 5.00-V ideal battery.

   A) When the current reaches half its maximum value, at what rate is magnetic energy being stored in the inductor?

   Since \( I = \frac{1}{2} I_{\text{max}} \), both \( R \) and \( L \) have half the battery voltage: 2.5 V.

   So \( P = I_L V_L = \frac{1}{2} I_{\text{max}} \cdot \frac{1}{2} V_B = \frac{1}{4} \frac{V_B^2}{R} \)

   \[ = 0.125 \text{ W} \]

   B) When the current reaches half its maximum value, what is the total power that the battery supplies?

   \[ P = I V_B = \frac{1}{2} \frac{V_B^2}{R} = 0.25 \text{ W} \]
4. (15 points) For the most part, the voltage in your house is 120 V. But some things that require very high power (dryers, stoves, AC) may require 220 V instead. In class, we calculated the voltage provided by a car's generator. In this problem, estimate the size of a loop (i.e., the area) required if you needed to generate this higher voltage. Recall that \( V(t) = V_{\text{MAX}} \sin(\omega t) \) and use \( B = 0.5 \text{T} \) and \( f = 60 \text{ Hz} \).

\[
V = \omega BA, \quad \Rightarrow \quad A = \frac{V}{\omega B}
\]

\[
A = \frac{\left( \frac{220V}{2\pi \times 60 \frac{\text{rad}}{\text{sec}} \times 0.5 \text{T}} \right)}{2 \pi}
\]

\[
\approx \frac{5}{2 \pi} \text{ m}^2
\]

\[
= 1.17 \text{ m}^2 = (1.08 \text{ m})^2
\]
5. **(25 points)** You can consider the liquid outer core of the Earth to be a solenoid of radius $1.2 \, Mm$ and length $3 \, Mm$ (but with only one turn – so $N = 1$). The strength of the magnetic field in the center is $2.5 \, mT$.

A) Calculate the current in this "solenoid".

\[
B = \mu_0 \frac{I}{2} \rightarrow
\]

\[
I = \frac{B l}{\mu_0} = \frac{(2.5 \times 10^{-3} \frac{T}{m})(3 \times 10^6 \, m)}{12.5 \times 10^7 \, A} \]

\[
= 6 \times 10^9 \, A
\]

B) Calculate the self inductance of the Earth’s liquid outer core.

\[
L = \mu_0 \frac{n^2}{\mu_l} = \frac{\mu_0 \pi r^2}{l}
\]

\[
= \frac{(12.5 \times 10^{-7} \frac{T}{A})(3 \times 10^6 \, m)(1.2 \times 10^6 \, m)}{3 \times 10^6 \, m} = 1.89 \frac{J}{A^2}
\]

C) Calculate the magnetic energy stored in Earth’s liquid outer core.

\[
U = \frac{1}{2} L I^2 = \frac{1}{2} \left(1.89 \frac{J}{A^2}\right)(6 \times 10^9 \, A)^2
\]

\[
= 3.4 \times 10^{19} \, J
\]

(about what the US uses in $2\frac{1}{2}$ years)