Some stuff to think about: Read all the questions first and do the ones you find easier first. Feel free to ask if something is unclear or you feel you need other information - I won't tell you how to work the problem, but I'm happy to clarify. TELL ME WHO YOU ARE. Above all, relax and say your mantra. The time has come, the walrus said, to speak of many things.

1. Consider the figure in Problem 20.10 (page 793), which shows a wire carrying a current, with a square loop nearby. Imagine that the loop moves toward the wire, passes over and moves away on the other side, and consider three time periods: 1) as the square loop moves toward the wire; 2) as the square loop passes over the wire; and 3) as the square loop moves away on the other side.

I call the initial flux positive

(8 points) A) Sketch a graph of the magnetic flux through the loop as a function of time. Indicate on the sketch the three time periods listed above.

(4 points) B) Sketch a graph of the induced voltage in the square loop. Indicate on the sketch the three time periods listed above.

(24 points) C) For the three time periods, indicate the direction of the current INDUCED in the square loop (CW or CCW). Explain your choice.

1) The initial flux is in and increasing, so the induced flux will be out. So the current will be CCW.

2) The external flux (in) will decrease to zero and then increase (out). To oppose this, the induced current will be CW.

3) The external flux will be out and decreasing, so the induced flux will be out. So the induced current will be CCW.
2. (12 points) In class, we calculated the voltage generated by a car alternator. In this problem, estimate the size of a loop (i.e., the area) required to generate the max voltage in your household (i.e., 110 V). Recall that \( V(t) = V_{\text{max}} \sin(\omega t) \) and use \( B = 2T \) and \( f = 60 \text{ Hz} \).

\[
V_{\text{max}} = BA\omega.
\]

\[
\omega = 2\pi f \approx 360\%.
\]

\[
A = \frac{V_{\text{max}}}{B\omega} = \frac{110 \frac{\text{V}}{\text{A}}}{\frac{2}{\text{A} \cdot \text{m}} \cdot 360\%} \text{ m}^2
\]

\[
= \frac{110}{720} \text{ m}^2 = 0.15 \text{ m}^2
\]
3. In class, we estimated the self-inductance of a circular loop by assuming that the strength of the field over the loop was the same as the value of the field at the center. (Recall that the result was \( L = (\mu_0 \pi R) / 2 \)). Do the same for a square loop.

(8 points) A) That is, estimate the self-inductance of a square loop of wire, assuming that the field across the area of the loop is the same as the value at the center. (HINT: Recall that, by definition, \( \Phi_M = LI \).

\[
B = 4B_{\text{each}} = 4 \cdot \frac{\mu_0 I}{2 \pi (d/2)} = \frac{4}{\pi d} \mu_0 I
\]

\[
\Phi_M \approx BA = \left( \frac{4 \mu_0 I}{\pi d} \right) \cdot d^2 = \left( \frac{\mu_0 4d}{\pi} \right) I
\]

So, by definition of \( L \), \( L = \left( \frac{\mu_0 4d}{\pi} \right) I \)

(Note: The simplest approximation, \( L \approx \mu_0 d \) is basically the same.)

(8 points) B) Use your expression to get a numerical value for the self inductance of a loop of wire big enough to go around a house.

An average is about 1500 (ft)^2. So each side is about \( \sqrt{1500 \text{ ft}^2} \approx 40 \text{ ft} \approx 13 \text{ m} \).

So \( L \approx \left( 4 \times 10^{-7} \right)^2 \cdot (4)(13 \text{ m}) \)

\[
L \approx 2 \times 10^{-5} \frac{J}{A^2}
\]
4. Consider an AC voltage generator in series with an ideal inductor (FIG 21.7a), p. 809). Recall that, in this condition, if the voltage is \( V(t) = V_{\text{max}} \sin(\omega t) \), then the current satisfies \( I(t) = \left( \frac{V_{\text{max}}}{\omega L} \right)(-\cos(\omega t)) \).

(16 points) A) Show (however you like) that the average power delivered to the circuit is zero. (HINT: It is similar, but NOT identical, to what we did with the capacitor.)

\[
P_{\text{av}} = IV = \frac{V_{\text{max}}^2}{2\omega L} \sin(\omega t) \cos(\omega t) = -\frac{V_{\text{max}}^2}{2\omega L} \sin(2\omega t)
\]

the \( \sin \) function, average: \( I^2 = 0 \)

(12 points) B) The denominator above is the inductive reactance: \( X_L = \omega L \). Explain PHYSICALLY why the reactance has the form it does (i.e., why does the reactance increase as \( \omega \) increases; similarly for \( L \)).

Since \( \phi = LI \), \( \Delta \phi = L \Delta I \), so the larger \( L \) is, the larger the \( \Delta \phi \) is. From FARADAY's LAW, \( V_{\text{ind}} \) (which opposes the flux change) is proportional to \( L \). If \( \omega \) is large, then \( T = \left( \frac{2\pi}{\omega} \right) \) is small. So \( V_{\text{ind}} = \frac{\Delta \phi}{\Delta t} \sim \frac{\Delta \phi}{T} \). If \( T \) small, \( V_{\text{ind}} \) which opposes the changing \( V_i \) is large.
5. (8 points) When I insert my credit card into a card reader (to, for example, buy gas), the computer tells me to “remove the card quickly”. Use the PHYSICAL ideas we developed in class to explain why it’s necessary to remove the card quickly rather than slowly. (HINT: The dark stripe on the back consists of magnetic materials). (OTHER HINT: The answer “so the computer can read your credit card number”, while correct, is hopelessly insufficient).

This is related to the example of waving a magnet over a loop of wire. As the flux changes, there will be an induced voltage. If the flux changes “quickly”, the induced voltage will be large, thus easier for the computer to read. 

\[ V = \frac{\Delta \Phi}{\Delta t} \]  

\[ n \cdot V = \nabla \cdot B \cdot L \]